Tropicalization in Convex Optimization

Semidefinite Optimization · Algebraic Geometry

Tropical geometry studies polynomials in the semiring $(\mathbb{R} \cup \{-\infty\}, \oplus, \otimes)$ with operations

$$x \oplus y = \max(x, y),$$
$$x \otimes y = x + y,$$

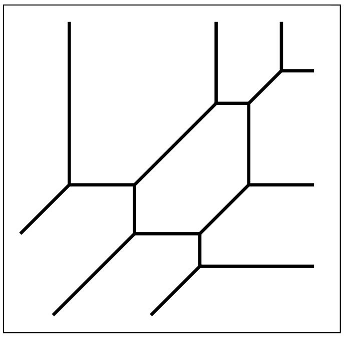
and neutral elements $-\infty$ and 0. Many algebraic and geometric objects have tropical equivalents. For example, the *tropicalization* of the polynomial $f = x^3 + 2xy + y^4$ is

$$\text{trop}(f) = \max(x + x + x, 2 + x + y, y + y + y + y).$$

The analogue of an algebraic curve $V(f) = \{x \mid f(x) = 0\}$ is defined as

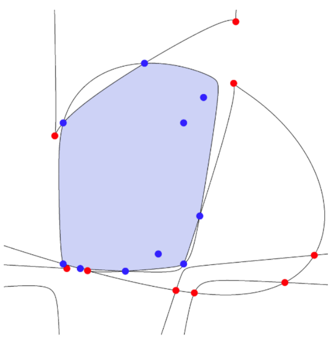
$$V(\text{trop}(f)) = \{x \mid \text{trop}(f) \text{ is not differentiable in } x\},$$

i.e., it consists of the points x in which the maximum is attained in at least two terms.

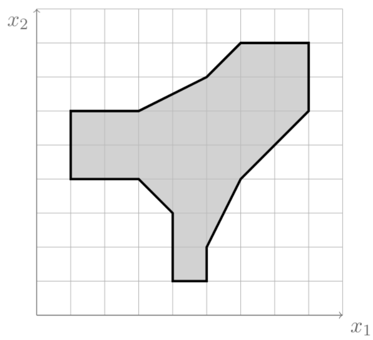


A cubic tropical curve

Often, the *tropicalization* of an object is easier to study, and can be used to find properties of the original objects. Recently, *spectrahedra*, the key objects in *semidefinite optimization*, as well as the related *sums-of-squares* and *moment* cones were tropicalized [Allamigeon et al. 2020, Blekherman et al. 2022]. The tropicalized cones are polyhedral, often easily described, and were used to find limits to the strength of semidefinite programming.



A Spectrahedron (blue)



A tropical Spectrahedron

Main tasks

Work out the basics of tropical geometry, and explain the tropicalization of spectrahedra.

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