Symmetric Term Sparsity

Graph Theory · Optimization · Algebra

A *polynomial optimization problem* is a problem of the form

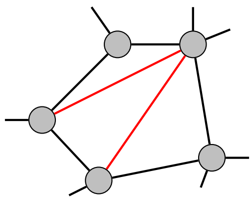
$$\begin{aligned} \min\ & f(x) \\ \text{s.t.}\ & g_i(x) \geq 0 \quad \text{for } i = 1, \dots, m, \\ & x \in \mathbb{R}^n, \end{aligned}$$

where f and the g_i's are polynomials in $\mathbb{R}[x_1, \dots, x_n]$. We can approximate such problems with semidefinite programming using the *moment-SOS-hierarchy*, which aims to approximate nonnegative polynomials on the feasible set using sums-of-squares:

$$\max \lambda \colon f - \lambda = \mathrm{SOS}_0 + \sum_{i=1}^{m} g_i \mathrm{SOS}_i,$$

where SOS_i are (globally nonnegative) polynomials of the form $\sum_j p_j^2$ for some polynomials p_j. If we limit the maximum degree of all appearing polynomials (thus obtaining a *level* of the approximation hierarchy), we obtain a computational approach we can implement using convex optimization methods.

While many strong theoretical results for the hierarchy exist, in practice the hierarchy often grows too quickly to be tractable on modern computers. In order to reduce this computational burden, ideas exploring the *sparsity* or *symmetry* of the original polynomial optimization problem were introduced. One successful approach [Jie Wang, Haokun Li, Bican Xia, 2019] exploits the *term sparsity of the problem*: We build a graph G with vertices being the set of all monomials, and connecting two monomials with an edge if their product is a monomial which appears in the polynomials f, g_i defining the original problem. Afterwards, an iterative refinement procedure is applied to the graph: we repeatedly apply a *chordal extension* and a *support extension* to the graph. A chordal extension adds additional edges to a graph until it is *chordal*, i.e., any cycle in G of length longer than 3 has a *chord*, an inner edge:



An excerpt of a graph and one of its chordal extensions in red.

The support extension relies on the structure of the original problem. The resulting sequence of denser and denser graphs can be used to build more efficient optimization models, in practice often significantly reducing the effort needed to solve them.

Main tasks

Read the literature and describe in detail how one can exploit term sparsity in polynomial optimization. Then explore the interplay of symmetries in the original problem and its term sparsity. Can we find a sparse chordal extension, which does not change the automorphism group of the graph? Can we find it efficiently, and exploit it computationally?

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