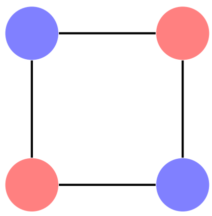
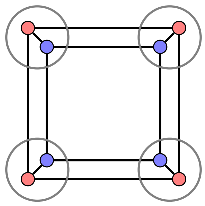
Correspondence Coloring

Combinatorial Optimization · Graph Theory

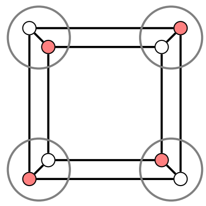
Given a graph $G = (V, E)$, a *k-coloring* of G assigns (up to) k colors to the vertices of G, such that no two adjacent vertices are colored the same. Checking whether a graph has a valid k-coloring reduces to checking whether the *k-cover* of G has a stable set of size k. This graph is obtained by *blowing up* every vertex of G with a k-clique:



2-coloring of C_4

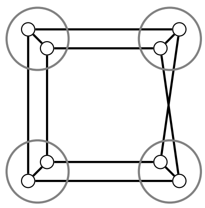


2-cover of C_4



Stable set in the cover

A new variant of the graph coloring problem known as *correspondence coloring* (or *DP-coloring*) was introduced by Dvořák and Postle in 2018. Here we are given, in addition to the graph $G = (V, E)$, an integer k, and (partial) matchings $M_e \subseteq K_{k,k}$ for every edge $e \in E$. The matchings M_e fix how the colors of the different vertices *correspond* to each other, and may add twists to the cover graph:



2-cover with a twist

We say that G is *k-M-colorable* if the correspondence graph has a stable set of size k. More generally, we call a graph *k-correspondence-colorable*, if G is k-M-colorable for every choice of matchings M. It can be shown that C_4 is 3-correspondence-colorable, i.e., as soon as we have three choices for each vertex we can always find a valid coloring, no matter how the colors correspond to each other.

Main tasks

Give an overview of the literature on the correspondence coloring. Implement an algorithm to check for k-M-colorability based on integer or semidefinite programming, and explore ideas for the harder question of k-correspondence-colorability.

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