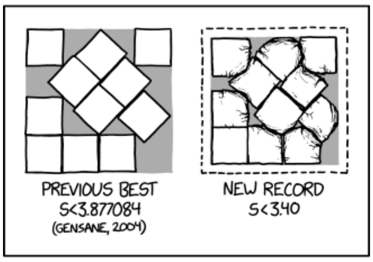
Geometric Packing Problems

Geometric Optimization · Linear/Semidefinite Programming

The *square packing problem* is a notoriously difficult geometric problem: What is the minimum side length of a square, in which we can fit N non-overlapping unit squares? The (few) known optimal solutions of the problem often look counter-intuitive and irregular.



I'VE SIGNIFICANTLY IMPROVED ON THE SOLUTION TO THE N=11 SQUARE PACKING PROBLEM BY USING A HYDRAULIC PRESS.

Even "easier" variants, such as packing unit circles into a square, a larger circle, or surface of a higher dimensional sphere are largely unsolved. The main difficulty is the high number of *local optima* of the problems.

To get around this, various *convexifications* of the problems have been proposed, based on linear or semidefinite programming. Instead of solving the original problem, we can relax it to a convex optimization problem, and use convex duality theory to prove bounds on the packing problems. Depending on the approach, *symmetry-breaking* or *symmetry-reduction* methods (based on harmonic analysis) are crucial to obtaining strong and computationally feasible optimization problems.

Main tasks

Explore some modeling, convexification and solution approaches from the literature for a variant of this problem. Depending on interests, this may involve linear programming, semidefinite programming and harmonic analysis. Implementing a bound computationally is an option.

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