Geometric Packing Problems

Geometric Optimization · Linear/Semidefinite Programming

The square packing problem is a notoriously difficult geometric problem: What is the minimum side length of a square, in which we can fit \( N \) non-overlapping unit squares? The (few) known optimal solutions of the problem often look counter-intuitive and irregular.

Even "easier" variants, such as packing unit circles into a square, a larger circle, or surface of a higher dimensional sphere are largely unsolved. The main difficulty is the high number of local optima of the problems.

To get around this, various convexifications of the problems have been proposed, based on linear or semidefinite programming. Instead of solving the original problem, we can relax it to a convex optimization problem, and use convex duality theory to prove bounds on the packing problems. Depending on the approach, symmetry-breaking or symmetry-reduction methods (based on harmonic analysis) are crucial to obtaining strong and computationally feasible optimization problems.

Main tasks

Explore some modeling, convexification and solution approaches from the literature for a variant of this problem. Depending on interests, this may involve linear programming, semidefinite programming and harmonic analysis. Implementing a bound computationally is an option.

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