J-ideals of sets of matrices

Let $A \in \mathbb{Z}^{n\times n}$, p a prime and $k \in \mathbb{N}$. We define the *p^k-ideal of A* as

$$N_{p^k}(A) = \{f \in \mathbb{Q}[x] \mid f(A) \in p^k\mathbb{Z}^n \times n\}.$$

The publications [1, 2] contain a thorough study of these ideals including an algorithm to compute their generating sets (which is implemented in SageMath).

The aim of this thesis is to study p^k-ideal of sets of matrices. For set $\mathcal{A}$ of matrices in $\mathbb{Z}^{n\times n}$, we define its p^k-ideal as

$$N_{p^k}(\mathcal{A}) = \bigcap_{A\in\mathcal{A}} N_{p^k}(A).$$

In particular, the following problems are open:

1. Necessary or sufficient conditions on $\mathcal{A}$ for $N_{p^k}(\mathcal{A})$ to be equal to $N_{p^k}(B)$ for a matrix $B \in \mathbb{Z}^{n\times n}$.
2. Necessary or sufficient conditions on $\mathcal{A}$ for $N_{p^k}(\mathcal{A})$ to be a principal ideal?
3. Necessary or sufficient conditions on $\mathcal{A}$ to be $N_{p^k}(\mathcal{A})$ trivial?

More generally, these question could be studied for matrices in $R^{n\times n}$ where R is a principal ideal domain (or even more general). The results in [1, 2] hold for principal ideal domains. For a domain R with quotient field K, an ideal J of R, and a matrix $A \in R^{n\times n}$ we define the *J-ideal of A* as

$$N_J(A) = \{f \in K[x] \mid f(A) \in J^{n\times n}\}.$$

Analogously, the J-ideal of a set of matrices is the intersection of the individual J-ideals.

[1] Clemens Heuberger and Roswitha Rissner, *Computing J-ideals of a matrix over a principal ideal domain*, Linear Algebra Appl. **527** (2017), 12–31.

[2] Roswitha Rissner, *Null ideals of matrices over residue class rings of principal ideal domains*, Linear Algebra Appl. **494** (2016), 44–69.

For further details and literature talk to

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