**J-ideals of sets of matrices**

Let $A \in \mathbb{Z}^{n \times n}$, $p$ a prime and $k \in \mathbb{N}$. We define the $p^k$-ideal of $A$ as

$$N_{p^k}(A) = \{ f \in \mathbb{Q}[x] \mid f(A) \in p^k \mathbb{Z}^{n \times n} \}.$$ 

The publications [1, 2] contain a thorough study of these ideals including an algorithm to compute their generating sets (which is implemented in SageMath).

The aim of this thesis is to study $p^k$-ideal of sets of matrices. For set $\mathcal{A}$ of matrices in $\mathbb{Z}^{n \times n}$, we define its $p^k$-ideal as

$$N_{p^k}(\mathcal{A}) = \bigcap_{A \in \mathcal{A}} N_{p^k}(A).$$

In particular, the following problems are open:

1. Necessary or sufficient conditions on $\mathcal{A}$ for $N_{p^k}(\mathcal{A})$ to be equal to $N_{p^k}(B)$ for a matrix $B \in \mathbb{Z}^{n \times n}$.

2. Necessary or sufficient conditions on $\mathcal{A}$ for $N_{p^k}(\mathcal{A})$ to be a principal ideal?

3. Necessary or sufficient conditions on $\mathcal{A}$ to be $N_{p^k}(\mathcal{A})$ trivial?

More generally, these question could be studied for matrices in $R^{n \times n}$ where $R$ is a principal ideal domain (or even more general). The results in [1, 2] hold for principal ideal domains. For a domain $R$ with quotient field $K$, an ideal $J$ of $R$, and a matrix $A \in R^{n \times n}$ we define the $J$-ideal of $A$ as

$$N_J(A) = \{ f \in K[x] \mid f(A) \in J^{n \times n} \}.$$ 

Analogously, the $J$-ideal of a set of matrices is the intersection of the individual $J$-ideals.


For further details and literature talk to

**Roswitha Rissner**

📍 Raum N.2.25

📞 +43 463 2700 3149

✉️ roswitha.rissner@aau.at