Tightness of Siegel's lemma (in special cases)

Let $A \in \mathbb{Z}^{m\times k}$ with $k > m$ such that every entry has absolute value less than or equal to b for some $b \in \mathbb{N}$. Siegel's lemma [1, Lemma 1, Chapter 2] states that $\ker(A)$ contains a non-zero element $\boldsymbol{v} \in \mathbb{Z}^k$ with

$$\|\boldsymbol{v}\|_\infty \leq (kb)^{\frac{m}{k-m}}. \tag{1}$$

For matrices A with non-negative entries whose row sums all equal to n, an adaption of the result (1) yields an improved bound:

$$\|\boldsymbol{v}\|_\infty \leq \left\lfloor (n)^{\frac{m-u}{n-m}} \right\rfloor, \tag{2}$$

where u denotes the number of rows of A with exactly one non-zero entry [2, Lemma 5.2]. The aim of this thesis is to investigate the tightness of the latter bound (2). Can you construct examples which reach the upper bound or can you sharpen the bound further (possibly with additional assumptions)?

[1] Alan Baker, *Transcendental number theory*, second ed., Cambridge Mathematical Library, Cambridge University Press, Cambridge, 1990. MR 1074572

[2] Moritz Hiebler, Sarah Nakato, and Roswitha Rissner, *Characterizing absolutely irreducible integer-valued polynomials over discrete valuation domains*, J. Algebra **633** (2023), 696–721.

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