## Bounds for minors of certain block matrices

A $k$-*minor* of a matrix $B$ is a the determinant of a $k \times k$ submatrix of $B$. The largest absolute value of all minors of $B$ is denoted by

$$\Delta = \Delta(B) = \max\{|\det(M)| \colon M \text{ is a minor of } B\}$$

It is in general hard to determine $\Delta$ as there are $\binom{m}{k}\binom{n}{k}$ minors of $B$ given that $B$ is an $m \times n$ matrix. Computing all of them quickly becomes infeasible with growing values of $m$ and $n$. This is why we are interested in finding good upper bounds for $\Delta$. For general matrices we have Hadamard's inequality which states that

$$|\det(M)| \leq \prod_{i=1}^{k} \|\boldsymbol{m}_i\|$$

where $\boldsymbol{m}_i$ denotes the $i$-th column of $M$. Whenever all columns of $M$ are non-zero, the inequality is satisfied with equality if and only if the $\boldsymbol{m}_i$ are pairwise orthogonal.

We are interested in matrices of a special form. Namely, let $B \in \mathbb{Z}^{m \times n}$ be a block matrix of the form

$$B := \begin{array}{c} \begin{matrix} \overbrace{\hphantom{a_1 \cdots a_s}}^{s} & \overbrace{\hphantom{a_1 \cdots a_s}}^{s} & \hphantom{\ddots} & \overbrace{\hphantom{a_1 \cdots a_s}}^{s} & \overbrace{\hphantom{-I_r \;\; 1}}^{r+1} \end{matrix} \\ \begin{pmatrix} \boxed{\begin{matrix} \boldsymbol{a}_1 & \cdots & \boldsymbol{a}_s \\ -1 & \cdots & -1 \end{matrix}} & & & & \boxed{\begin{matrix} -I_r \\ 0 \cdots 0 \end{matrix}} & \boxed{\begin{matrix} \\ 1 \end{matrix}} \\ & \boxed{\begin{matrix} \boldsymbol{a}_1 & \cdots & \boldsymbol{a}_s \\ -1 & \cdots & -1 \end{matrix}} & & & \boxed{\begin{matrix} -I_r \\ 0 \cdots 0 \end{matrix}} & \boxed{\begin{matrix} \\ 1 \end{matrix}} \\ & & \ddots & & \vdots & \\ & & & \boxed{\begin{matrix} \boldsymbol{a}_1 & \cdots & \boldsymbol{a}_s \\ -1 & \cdots & -1 \end{matrix}} & \boxed{\begin{matrix} -I_r \\ 0 \cdots 0 \end{matrix}} & \boxed{\begin{matrix} \\ 1 \end{matrix}} \end{pmatrix} \begin{matrix} \left.\vphantom{\begin{matrix} a \\ 1 \end{matrix}}\right\} r+1 \\ \left.\vphantom{\begin{matrix} a \\ 1 \end{matrix}}\right\} r+1 \\ \\ \left.\vphantom{\begin{matrix} a \\ 1 \end{matrix}}\right\} r+1 \end{matrix} \end{array},$$

where $\boldsymbol{a}_1, \ldots, \boldsymbol{a}_s \in \mathbb{N}_0^r$ and $I_r$ is the $r \times r$-identity matrix.

The task of this thesis is to investigate upper bounds for $\Delta(B)$, potentially restricting the choices of $r$, $s$ as well as the entries of the column vectors $\boldsymbol{a}_1$, $\ldots$, $\boldsymbol{a}_s$. Can you find conditions which allow you to determine an upper bound that is tighter than Hadamard's inequality?

For further details and literature talk to

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