Distributed Collaborative State Estimation: Joint Observations for Reliable Autonomous Navigation in Swarms

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***Abstract*— This paper emphasis the role of Collaborative State Estimation (CSE) for reliable navigation and sensing in heterogeneous swarms of communicating robots. By using a state-of-the-art CSE algorithm, communication and maintenance of interdependencies is mainly needed for the moment of joint observations, while the credibility and performance of the distributed estimators remain closely to the statistically optimal centralized solution. In simulation we demonstrate the concept of sensor sharing to improve the localization accuracy and sensor relaying in case of sensor failure in an exploration scenario, rendering CSE as a key for reliable sensing in swarms of communicating robots.**

I. INTRODUCTION

Estimating states collaboratively among a group of communicating agents is a key to achieve precise and robust localization in challenging situations. Properties such as *Sensor Sharing* or indirect *Sensor Relaying* can be inherently established by processing joint observation between members of a group.

For the purpose of localization, common joint observations are local relative pose, position, range, bearing, or range and bearing measurements between two agents. Theoretically, any measurement that directly or indirectly observes any estimated state can be fused, meaning that observations do not have to be pair-wise and do not have to contain localization information.

This allows for a redundancy in a global scale as local sensor failures might be compensated by joint observation with other robots. Further, robots equipped with less accurate sensors can benefit from robots with more accurate sensors as shown e.g. by Roumeliotis and Rekleitis in [1]. In [2], Jung et al. have shown that agents receiving just joint relative position measurement with respect to other agents, (i) can navigate with respect to a common coordinate reference frame and (ii) those equipped with an Inertial Measurement Unit (IMU) can restore their 6-DoF pose.

On a local scale, complementary and different sensor modalities are typically used for precise and autonomous navigation of robots [3], [4]. Fusing inertial and camera information has proven well in so called Visual-Inertial Navigation System (VINS) (e.g. Geneva et al. [5]). In [6],

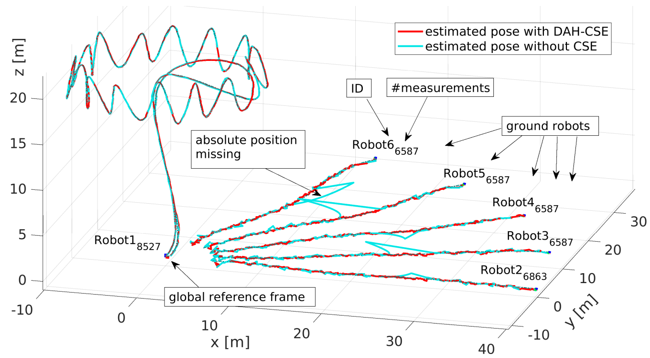

Fig. 1: Estimated trajectories of agents fusing joint relative position measurements using DAH-CSE (red) or not (cyan). Agent/robot A_1 is flying above the agents on the ground $\mathsf{A}_{\{2,\ldots,6\}}$. The agents receive IMU and absolute position measurements. The ground robots are suffering from sensor outages and dropped absolute position measurements leading to drifting estimates in case of not performing CSE (cyan). The problem is described in Section IV.

Brommer et al. open-sourced the Modular and Robust Sensor-Fusion (MaRS) framework allowing for efficiently fusing such sensors with a core state propagated by IMU measurements.

In this paper, we demonstrate that Collaborative State Estimation (CSE) can play an important role for reliable sensing in the mobile robotics sector. In a simplified exploration scenario, where ground robots are equipped with less accurate positioning sensors, that suffer from dropped sensor reading, and sensors failure, an additional robot (e.g. an Unmanned Aerial Vehicle (UAV)) equipped with a more accurate positioning is circulating above them and performs joint relative position updates with each of them.

CSE has been addressed in the past decades with the main ambitions to decouple the individual agents, while reducing communication and compute complexity [2], [7]–[14].

Rendering Collaborative State Estimation (CSE) distributed and scalable with number of agents in a team was a subject in the recent work of Jung and Weiss [14]. In simulation on a swarm of 20 communicating agents it was shown that processing relative position update allows each individual agent to estimate all 6-DoF while hidden states could converge and only six agents had access to absolute position information.

In this paper, we use this Distributed Approximated History (DAH) CSE approach to evaluate the impact of processing relative position measurements in case of faulty and unreliable sensors on ground agents. The simulation results show, that these agents can significantly improve the

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estimation accuracy at the cost of a slightly higher processing overhead, rendering CSE as novel approach to ensure reliable autonomous navigation in swarms.

II. Notation

The mean and covariance of multivariate random variable are defined as $\mathbf{X}_i \sim \mathcal{N}(\hat{\mathbf{x}}_i, \mathbf{\Sigma}_{ii})$. A right subscript specifies the agent's identifier $\{\mathsf{A}_i, i \in 1, \ldots, N\}$. The time indices of state variables are indicated by the right superscript, e.g. $\mathbf{X}^k$, denoting the state at the time $t(k) \equiv t^k$. Names of reference frames are capitalized and calligraphic, e.g. $\mathcal{I}$ for IMU. A coordinate vector ${}^{\mathcal{C}}_{\mathcal{C}}\mathbf{p}_{P_1}$ is read as ${}^{from}_{in}\mathbf{x}_{to}$. The operators $\boxplus$ and $\boxminus$ should emphasize that rotational in SO^3 and translational components in $\mathbb{R}^3$ have to be treated differently. Positions, velocities and biases are additive, e.g. ${}^{\mathcal{G}}\mathbf{p}_{\mathcal{I}} = {}^{\mathcal{G}}\hat{\mathbf{p}}_{\mathcal{I}} + {}^{\mathcal{G}}\tilde{\mathbf{p}}_{\mathcal{I}}$. Rotational errors are right-multiplicative, e.g. ${}^{\mathcal{G}}\mathbf{R}_{\mathcal{I}} = {}^{\mathcal{G}}\hat{\mathbf{R}}_{\mathcal{I}}(\mathbf{I}_3 + \left[{}^{\mathcal{G}}\tilde{\boldsymbol{\theta}}_{\mathcal{I}}\right]_{\times}) \in \mathrm{SO}^3$.

III. Problem Formulation

A swarm of N communicating agents equipped with an IMU as proprioceptive sensor and an exteroceptive sensor providing absolute position information (e.g. a Global Navigation Satellite System (GNSS) sensor), is navigating in space. Only one agent is able to measure the local relative position of other agents, e.g. by sensing the range and bearing angles or by a camera-based visual tag detection. Using the IMU as a strapped down propagation sensor makes the estimator independent on the underling kinematic motion model of the agent. Note, that CSE is not restricted to the aided-inertial estimation case, as theoretically any state propagation model can be used e.g. different odometry models for ground robots.

Each agent estimates it's IMU navigation state $\mathbf{X}_i$ using a Quaternion-based Error-State Extended Kalman Filter (Q-ESEKF) (e.g. [5])

$$\mathbf{X}_i = \left[{}^{\mathcal{G}}_{\mathcal{G}}\mathbf{p}_{\mathcal{I}}, {}^{\mathcal{G}}_{\mathcal{G}}\mathbf{v}_{\mathcal{I}}, {}^{\mathcal{G}}\mathbf{q}_{\mathcal{I}}, {}_{\mathcal{I}}\mathbf{b}_{\omega}, {}_{\mathcal{I}}\mathbf{b}_{a}\right]_i, i \in 1, \ldots, N, \tag{1}$$

with ${}^{\mathcal{G}}\mathbf{p}_{\mathcal{I}}, {}^{\mathcal{G}}_{\mathcal{G}}\mathbf{v}_{\mathcal{I}}$, and ${}^{\mathcal{G}}\mathbf{q}_{\mathcal{I}}$ as the position, velocity and orientation of the IMU $\mathcal{I}$ w.r.t. the global frame $\mathcal{G}$ (or navigation frame). ${}_{\mathcal{I}}\mathbf{b}_{\omega}$ and ${}_{\mathcal{I}}\mathbf{b}_{a}$ are the estimated gyroscope and accelerometer biases to correct the related IMU readings.

Initially, the agents' states can be seen as decoupled multivariate variables of a global swarm state $\mathbf{X}^k \sim \mathcal{N}(\hat{\mathbf{x}}^k, \mathbf{\Sigma}^k)$, with $\hat{\mathbf{x}}^k = \left[\hat{\mathbf{x}}^k_1; \; \ldots; \; \hat{\mathbf{x}}^k_N\right]$ the estimated values and, $\mathbf{\Sigma}^k = \left[\mathbf{\Sigma}^k_{i,j}\right]_{1 \leqslant i,j \leqslant N} \in S^n_+$ the uncertainties. In the beginning, agents are uncorrelated $\{\mathbf{\Sigma}_{i,j} = \mathbf{0} : i,j \in 1, \ldots, N, i \neq j\}$, while joint observation between agents, e.g. i and j, leads to cross-covariances $\mathbf{\Sigma}_{i,j} \neq \mathbf{0}$.

In a centralized CSE formulation, the entire swarm state is estimated leading to statistically optimal estimates, at the cost of compute and communication effort. To render CSE distributed among agents various exact and approximated approaches have been proposed e.g. [7], [11], [14], [15]. In this paper, we use DAH-CSE proposed by Jung and Weiss in [14] as it (i) requires communication just at the moment of joint observations, and (ii) maintenance effort for propagation and private observation is constant $\mathcal{O}(1)$.

Note, in CSE three different filter steps can be performed: (i) state propagation, (ii) private observation correcting and requiring just one agent's state, and (iii) joint observation referring to an arbitrary number of states.

A. State Propagation

Each inertial navigation state $\mathbf{X}_i$ is propagated forward using agent A_i's IMU samples containing noisy and biased linear acceleration ${}_{\mathcal{I}_i}\mathbf{a}_m{}^k = {}^{\mathcal{G}}\mathbf{R}_{\mathcal{I}_i}{}^k\mathbf{g} + {}_{\mathcal{I}_i}\mathbf{a}^k + {}_{\mathcal{I}_i}\mathbf{b}_a{}^k + {}_{\mathcal{I}_i}\mathbf{n}_a$ and angular velocity ${}_{\mathcal{I}_i}\boldsymbol{\omega}_m{}^k = {}_{\mathcal{I}_i}\boldsymbol{\omega}^k + {}_{\mathcal{I}_i}\mathbf{b}_{\omega}{}^k + {}_{\mathcal{I}_i}\mathbf{n}_{\omega}$ measurements, with $\mathbf{n}$ denoting the zero-mean white Gaussian noise. The nonlinear error-state IMU kinematic propagation function for from t^{k-1} to t^k is modeled as [3]

$$\mathbf{X}^k_i = f(\mathbf{X}^{k-1}_i, {}_{\mathcal{I}_i}\mathbf{a}_m{}^k, {}_{\mathcal{I}_i}\boldsymbol{\omega}_m{}^k) + \mathbf{n}^k_{\mathcal{I}_i}, \tag{2}$$

with the measurement noise $\mathbf{n}^k_{\mathcal{I}_i} = \mathcal{N}\left(0, \mathbf{R}^k_{\mathcal{I}_i}\right)$. This result after linearization and time discretization in the state transition matrix $\mathbf{\Phi}^{k|k-1}_i$ and process noise matrix $\mathbf{Q}^{k|k-1}_i$ [3] to propagate the state covariance matrix

$$\mathbf{\Sigma}^k_i = \mathbf{\Phi}^{k|k-1}_i \mathbf{\Sigma}^{k-1}_i (\mathbf{\Phi}^{k|k-1}_i)^{\mathsf{T}} + \mathbf{Q}^{k|k-1}_i. \tag{3}$$

For details, we would like to refer interested readers to the online documentation of OpenVINS [5].

Note that for DAH-CSE, $\mathbf{\Phi}^{k|k-1}_i$ has to be inserted into the history buffer $\boldsymbol{\mathcal{B}}_i$.

B. Private Absolute Position Observation

Absolute position measurements can be described by a nonlinear function

$$\mathbf{z}^k_{abs} = h_{abs}(\mathbf{X}^k_i) + \mathbf{n}^k_{abs} = {}^{\mathcal{G}}_{\mathcal{G}}\mathbf{p}_{\mathcal{I}} + \mathbf{n}^k_{abs}, \tag{4}$$

with the measurement noise $\mathbf{n}^k_{abs} = \mathcal{N}\left(0, \mathbf{R}^k_{abs}\right)$. In order to update the Q-ESEKF, the measurement function has to be linearized with respect to the error state $\widetilde{\mathbf{x}} = \mathbf{x} \boxminus \hat{\mathbf{x}}$ to obtain the measurement Jacobian

$$\mathbf{H}^k = \left.\frac{\partial h(\hat{\mathbf{x}}^k_i)}{\partial \widetilde{\mathbf{x}}^k}\right|_{\hat{\mathbf{x}}^{k(-)}}. \tag{5}$$

Now, we can perform the standard EKF update as follows

$$\mathbf{K}^k_i = \mathbf{\Sigma}^{k(-)}_{ii}(\mathbf{H}^k)^{\mathsf{T}}\left(\mathbf{H}^k\mathbf{\Sigma}^{k(-)}_{ii}(\mathbf{H}^k)^{\mathsf{T}} + \mathbf{R}^k\right)^{-1} \tag{6}$$

$$\hat{\mathbf{x}}^{k(+)}_i = \hat{\mathbf{x}}^{k(-)}_i \boxplus \mathbf{K}^k_i\left(\mathbf{z}^k - h(\hat{\mathbf{x}}^k_i)\right) \tag{7}$$

$$\mathbf{\Upsilon}^k = \left(\mathbf{I} - \mathbf{K}^k_i\mathbf{H}^k\right) \tag{8}$$

$$\mathbf{\Sigma}^{k(+)}_{ii} = \mathbf{\Upsilon}^k\mathbf{\Sigma}^{k(-)}_{ii} \tag{9}$$

Note that for DAH-CSE, $\mathbf{\Upsilon}^k$ has to be inserted into the history buffer $\boldsymbol{\mathcal{B}}_i$.

C. Joint Relative Position Observation

Private and joint observations are technically the same, while the later requires, in addition to the local state estimate, estimates from one or multiple other agents. In DAH-CSE, joint observations are processed on an interim master, which receives all required information from the other participating agents and sends them the corrected information back. The relative position measurements between two agents A_i and A_j can be described by a nonlinear function

$$\begin{aligned} \mathbf{z}^k_{rel_{fi,jg}} &= h_{rel}(\mathbf{X}^k_i, \mathbf{X}^k_j) + \mathbf{n}^k_{rel} \\ &= {}^{\mathsf{G}}\mathbf{R}_{I_i}{}^{\mathsf{T}} \left(-{}^{\mathsf{G}}_{\mathsf{G}}\mathbf{p}_{I_i} + {}^{\mathsf{G}}_{\mathsf{G}}\mathbf{p}_{I_j} \right) + \mathbf{n}^k_{rel}, \end{aligned} \tag{10}$$

with the measurement noise $\mathbf{n}^k_{rel} = \mathcal{N}\left(0, \mathbf{R}^k_{rel}\right)$.

The joint belief of participants, e.g. constituting of A_i's and A_j's belief is $\mathbf{X}_p^{\mathsf{T}} = \begin{bmatrix}\mathbf{X}_i^{\mathsf{T}} & \mathbf{X}_j^{\mathsf{T}}\end{bmatrix}$. The joint *a priori* covariance is $\mathbf{\Sigma}^{k(-)}_{pp} = \begin{bmatrix}\mathbf{\Sigma}_{ii} & \mathbf{\Sigma}_{ij} \\ \mathbf{\Sigma}_{ij}^{\mathsf{T}} & \mathbf{\Sigma}_{jj}\end{bmatrix}^{k(-)}$, where $\mathbf{\Sigma}^{k(-)}_{ij}$ is restored using correction factors inserted into the history buffers $\boldsymbol{\mathcal{B}}_{fi,jg}$ and the previously factorized cross-covariance terms [14].

As in Section III-B, the measurement function has to be linearized with respect to the joint error state

$$\mathbf{H}^k_p = \left.\frac{\partial h(\hat{\mathbf{x}}^k_p)}{\partial \tilde{\mathbf{x}}^k_p}\right|_{\hat{\mathbf{x}}^{k(-)}_p} = \begin{bmatrix}\mathbf{H}^k_i & \mathbf{H}^k_j\end{bmatrix}. \tag{11}$$

Now, we can again perform the standard EKF update on the stacked/joint state. In case of DAH-CSE, the stacked state needs to be split again after the update and the corrected belief including a correction term has to be sent to the participating agents. The correction terms for joint observations is $\mathbf{\Lambda}^k_{fi,jg} = \mathbf{\Sigma}^{k(+)}_{fii,jjg}\left(\mathbf{\Sigma}^{k(-)}_{fii,jjg}\right)^{-1}$ and has to be inserted into the history buffers $\boldsymbol{\mathcal{B}}_{fi,jg}$, respectively. Note that in DAH-CSE, non-participating agents are not directly benefiting from joint observations as it would be the case in centralized equivalent filter.

IV. Experiments

The experiments are simulated in a MATLAB framework, that allows to load existing datasets or to generate random trajectories. The exteroceptive measurements (private or joint observations) are generated based on the ground truth trajectory, the sensors calibration states and noise parameters. Finally, all measurements from all agents are sorted chronologically and are locally processed in a multi-instance manager. It is maintaining multiple filter instances, while communication between filter instances is handled locally. The following simplifications are made:

- system clocks, IMUs and exteroceptive sensors are synchronized across the team,
- extrinsic calibrations between exteroceptive sensors and the IMU on an agent are known and static,
- the exteroceptive measurement (observation) noise and the process noise are independent,
- each agent has a unique identifier,
- no physical interaction between agents (i.e. the motion of an agent does not affect the motion of other agents),
- the period of exteroceptive sensors is an integer multiple of the IMU period,
- communication range is larger than the sensing distance, and exchanged information between agents and sensor measurements arrive without delay.

Note that, sensor and communication delay can be compensated by introducing time sorted buffers for sensor measurements and estimates as proposed in [4].

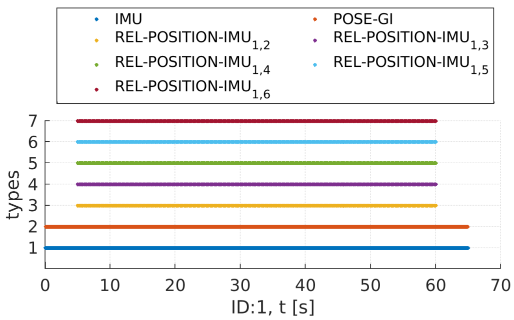
A. Scenario S_1

This scenario demonstrates that CSE can play an important role for reliable autonomous navigation in swarms by performing joint observation between an agent that is circulating in the air and agents (e.g. legged robots) on the ground that are randomly exploring the environment on the ground. For a direct comparison we performed the simulation with and without these relative position updates. To render a challenging exploration scenario, agents on the ground experience random message drops and temporary sensor outages. Further, the absolute position measurements of ground robots suffers from higher noise and lower rates, while the agent in the air obtains highly accurate absolute position information at higher rates. As described in III, all agents use noisy and biased IMU samples for the state propagation. An overview about the used simulation parameter can be found in Table I.

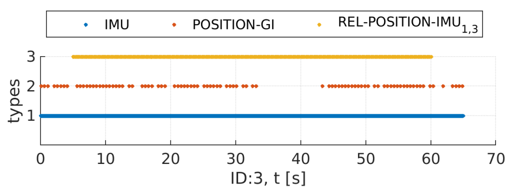
Further, the states were initialized wrongly to demonstrate the self-calibration capabilities and emphasize the state convergence. Table II summarize both, the Average Root Mean Square Error (ARMSE) and the total filter execution times for the individual filter steps with and without using joint relative position updates. As joint observations require a CSE fusion approach a computational overhead is expected. These relative observations are fused using DAH-CSE [14], requiring communication only for those observations and maintenance cost for interdependencies between agents. Processing joint observation reduces the ARMSE of all agents, while ground robots $\mathsf{A}_{f2,\ldots,6g}$ are suffering from inaccurate and unreliable absolute position measurements, can drastically improve their absolute position estimate from average ARMSE of $0.53\,\mathrm{m}$ to $0.1\,\mathrm{m}$ by an increased total filter execution time from $8.14\,\mathrm{s}$ to $9.22\,\mathrm{s}$. Using DAH-CSE, the interim master is mainly processing the joint update. In Figure 2a, measurements processed by A_1 (interim master) are shown. Relative position measurements are processed between $t = 5\,\mathrm{s}$ and $t = 60\,\mathrm{s}$ at a rate of $5\,\mathrm{Hz}$ with the agents $\mathsf{A}_{f2,\ldots,6g}$ and cause approximately $32\,\%$ of the total execution time of A_1. One remarkable feature of DAH-CSE is that it barely increasing the processing time of other filter steps (propagation and private observations) on all agents. For private update the increase is barely visible and should not exist theoretically. Note that the same holds for the propagation step, if the used correction history buffer is sufficiently large. In our experiments we set it to 250 ms. As the the joint observations ended at $t = 60\,\mathrm{s}$, the agents had to forward propagate their factorized cross-covariances that causes an computational overhead. In Figure 2b the measurements processed by agent A_3 are shown, which suffers from sporadically dropped absolute position measurements and a sensor outage between $t = 35\,\mathrm{s}$ and $t = 45\,\mathrm{s}$. In Figure 3b it can seen that those missing measurements drastically decrease the estimation accuracy, leading to an position estimation error of approximately $8\,\mathrm{m}$ on the x-axis. On the

Parameter	Value	Parameter	Value
Num. ground robots	5	abs. pos. rate (Air, Ground)	$10[Hz], 2[Hz]$
Duration	$65[s]$	Noise abs. pos. (Air, Ground)	$0.1[m], 0.5[m]$
IMU rate	$200[Hz]$	abs. pos drop rate (Air, Ground)	$0[\%], 20[\%]$
accelerometer noise	$0.01[m/s^2]$	abs. pos. off-time (Air, Ground)	$0[s], 10[s]$
gyroscope noise	$0.001[rad/s]$	relative pos. rate	$5[Hz]$
acc. rate random walk	$0.005[(m/s)/s/\sqrt{s}]$	relative pos. noise	$0.1[m]$
gyr. rate random walk	$0.0005[rad/s/\sqrt{s}]$	rel. pos drop rate	$0[\%]$

TABLE I: Simulation parameters of scenario S_1.

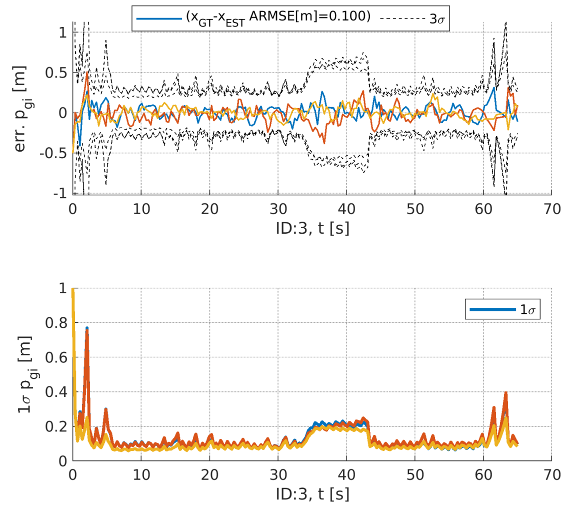


(a) Measurements processed by agent A_1.

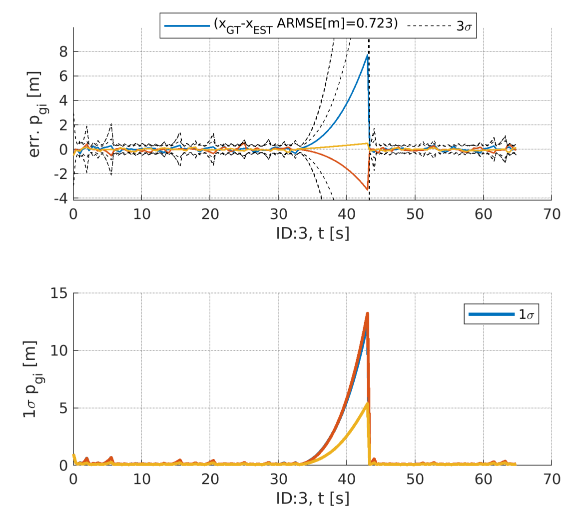


(b) Measurements processed by agent A_3.

Fig. 2: Scenario S_1: Shows the measurements processed on agent A_1 and A_3, while A_3's absolute position measurements (*POSITION-GI*) suffer from sporadic drop messages and a sensor outage between $t = 35$ and $t = 45$. As depict in Table II, joint relative observations with A_1 can compensate those outages and allows for accurate navigation.



(a) Estimated position error of A_3 receiving joint relative position updates.



(b) Estimated position error of A_3 not receiving joint updates.

Fig. 3: Scenario S_1: Shows the RMSE of agent A_3's estimated position (top) and uncertainty (bottom) with and without obtaining joint relative position observations. The graphs are held in blue, red and yellow for position x, y, z. As shown in Figure 2b, between $t = 35$ and $t = 45$, no private absolute position measurement are obtained. In case of joint observations, this leads to a slightly increased estimation error and increased uncertainty (the error remains bounded). Not obtaining them, causes the IMU state to drift unbounded until private observations are available again leading to an ARMSE of $0.723\,\mathrm{m}$ compared to $0.1\,\mathrm{m}$.

other hand, as depict in Figure 3a, by processing relative position observations with A_1, A_3 is able to keep bounded position estimates. It is noticeable that the uncertainty is increased when no joint observations are obtained between $t = 0\,\mathrm{s}$ and $t = 5\,\mathrm{s}$, $t = 35\,\mathrm{s}$ and $t = 45\,\mathrm{s}$, and $t = 60\,\mathrm{s}$ and $t = 65\,\mathrm{s}$. Summarized, the most important finding is that the ground agents can drastically improve their pose estimate by processing joint relative observation, while the filter execution time slightly increases.

V. Conclusion

Reliable and robust state estimation is a key to achieve precise localization in challenging situations. Typically, complementary and different sensor modalities are used. In this paper, we show that it can be achieved requiring communication between distributed estimators. Our simulation results show, that fusing joint observations using Collaborative State Estimation (CSE) reduces significantly the estimation error in case of faulty sensors at the cost of a slightly higher processing overhead. This renders CSE as novel approach to ensure reliable autonomous navigation in swarms.

		p_{gi} [m]	v_{gi} [m/s]	q_{gi} [deg]	b_a [m/sˆ2]	b_w [rad/s]	execution time [s]			
ID	REL TYPE	ARMSE	ARMSE	ARMSE	ARMSE	ARMSE	t_{prop}	t_{priv}	t_{joint}	t_{total}
1	Rel. pos	0.045	0.06	0.21	0.0002	0.004	8.58	0.83	4.4	13.82
2	Rel. pos	0.11	0.12	1.6	0.0001	0.007	8.22	0.1	0.006	8.32
3	Rel. pos	0.09	0.11	2.34	0.0001	0.01	8.25	0.09	0.006	8.35
4	Rel. pos	0.1	0.12	2.93	0.0002	0.01	8.19	0.09	0.006	8.29
5	Rel. pos	0.1	0.12	1.6	0.0002	0.007	8.18	0.09	0.006	8.27
6	Rel. pos	0.1	0.12	3.3	0.0002	0.011	8.18	0.088	0.006	8.27
1	None	0.05	0.07	0.28	0.0004	0.005	7.93	0.81	0	8.75
2	None	0.36	0.17	1.67	0.0001	0.007	7.93	0.09	0	8.02
3	None	0.75	0.23	2.49	0.0001	0.01	7.93	0.09	0	8.02
4	None	0.32	0.16	3.44	0.0002	0.01	7.93	0.09	0	8.02
5	None	0.64	0.2	1.97	0.0002	0.007	7.93	0.09	0	8.02
6	None	0.58	0.2	3.36	0.0002	0.011	7.93	0.09	0	8.02

TABLE II: Scenario S_1: ARMSE of the agents' navigation states defined in Equation (1) and total execution time of different filter steps (propagation, private updates and joint updates), processing relative position measurements between agent A_1 and the other agents $\mathsf{A}_{\{2,\ldots,6\}}$ or not.

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