A Critical Edition of Popper's Work on Logic

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Karl Raimund Popper's work on formal logic [1, 2, 3, 4, 5, 6, 7], written and published in the 1940s, is not as widely known as it deserves. Only very few detailed investigations into the philosophical and technical aspects of these articles have been published [8, 11, 12, 13, 14]. In view of the extensive literature on almost every other aspect of Popper's philosophy, this is in itself noteworthy, especially in view of the fact that decidedly logical arguments are at the core of Popper's philosophy of science.

There are various reasons which may explain this scholarly neglect. Popper himself disavowed this line of research, later classifying it as a poorly worked out rediscovery of natural deduction. The articles also contained technical errors that were pointed out by reviewers, something which discouraged Popper from continuing this work. But despite all this his articles contain ideas which merit a more detailed study. In order to make this part of Popper's legacy more available to other philosophers and logicians, we (Peter Schroeder-Heister, Thomas Piecha and David Binder) are currently preparing a critical edition of these articles together with a wealth of additional material from the Karl Popper-Sammlung of the university of Klagenfurt. This book will contain:

- Newly typeset versions of the published articles, together with errata, editorial history and introductions.
- All the original reviews written by Kleene, Curry, Beth, Ackermann, Hasenjaeger and McKinsey.
- Unpublished manuscripts written in preparation of the published articles and containing entirely new material relating to these articles.
- Correspondence. Popper discusses, clarifies and amends the content of these articles in correspondence with other logicians and mathematicians such as Carnap, Bernays, Quine, Forder and Brouwer.

Having studied thousands of unpublished pages from the collection of the Karl Popper-Sammlung in Klagenfurt, I will present an overview of the history of Popper's investigations into logic in the 1940s and how it can be reconstructed from the material we plan to include in the book. The following is a historical overview of Popper's preoccupation with formal logic.

Vienna. Popper probably first got in contact with logic and the foundations of mathematics by enrolling in a course of Hans Hahn in 1922 in which Principia

Mathematica was part of the curriculum. He soon got in contact with members of the Vienna circle, among them Gödel and Carnap, and in 1934 he met Tarski who had a profound influence on Popper's views on logic. We do know about the impression that Tarski's analysis of truth made on him, and in a letter written in 1943 he calls himself a "disciple of Tarski" and mentions that he helped Tarski in the translation of "Über den Begriff der logischen Folgerung" into German.[1] There is little written testimony about his views on formal logic during the Vienna years, due to the lack of publications on formal logic and the poor archival situation regarding the time before his departure to New Zealand.

Christchurch. In 1937 Popper had to flee from Austria and found employment as a lecturer at the University of Canterbury in Christchurch, New Zealand. Part of his teaching duties was a course in logic for philosophers. He was not content with the available logic textbooks suitable for philosophers and planned to write a logic textbook in ∼1937/38. The three people whom Popper discussed logical problems with during his time in Christchurch are, as far as we can see, John Findlay, Henry George Forder and Rudolf Carnap. The evidence for Findlay, who taught at the University of Otago at the time, is rather slim and rests on (1) handwritten remarks on a paper that is likely to be an early version of [9], and (2) the fact that Popper discussed that article with Paul Bernays in 1946. With Forder, a professor of mathematics at Auckland University College, on the other hand, the situation is clear since there is an extensive correspondence from February 1943 to July 1945 (23 letters in total). They discuss university politics but also problems in the philosophy of mathematics, logic and quantum physics. It is in these letters that Popper mentions for the fist time his conception of logic as a "meta-propositional calculus"; a particular interpretation of the inequations of boolean algebra. The contact with Carnap is through exchange of letters, averaging about three letters per year. Every time Carnap finishes another book, Introduction to Semantics in 1942 and Formalization of Logic in 1943, he sends a copy to Popper who replies with questions and sometimes long sheets of comments. Carnap is certainly, together with Tarski, the one person who inspired most of the logical investigations Popper undertook during that time. Remarks in letters and published and unpublished articles show that it is through reading Carnap that he found the problems that he tried to solve. In 1943 Popper writes a series of articles on boolean algebra, at least one of which he intended to publish in the Journal of Symbolic Logic.[2] They are called "Extensionality in a Rudimentary Boolean Algebra", "An Elementary Problem of Boolean Algebra", "Completeness and Extensionality of a Rudimentary Boolean Algebra", "Postulates for Boolean Algebra" and "Simply Independent Postulates for Boolean Algebra". Forder supported Popper by proofreading his typoscripts and by lending him articles that were not available in Christchurch, most importantly Huntington's [10] on which much of the development in Popper's articles is based.

London. In 1946 Popper gets a position at the London School of Economics and moves back to Europe. For reasons that are still not clear to us, he met with Bernays in Zürich in December 1946. During discussions, Bernays proposed to publish an article together with Popper who eagerly accepted and set

[1] Letter from Popper to H. G. Forder, May 7th 1943. Karl-Popper-Sammlung (KPS) 296, 15.

[2] In the LaTeX-version that we work with, these articles take up about 100 pages. They are from KPS 12,3; 12,4; 12,5; 16,13.

himself to work in the first months of 1947. He finished the article, entitled "On Systems of Rules of Inference" by March 3rd and sent a copy of the manuscript to Bernays.[3] The reason why the article never got published is unclear, but it seems that Bernays was not in full agreement with Popper regarding some of the arguments of the article. The content of this article is already quite close to the content of [2] and [3], but contains significant material that was omitted in those later articles. Among other things, it contains an explicit comparison with Tarski's system [15] and a criterion for the "purity" of inference rules[4]. Popper wrote on the distinction between derivation and demonstration in three unpublished drafts, written some time between the completion of "On Systems of Rules of Inference" and the writing of [3]. One of them is untitled; the other two are called "Derivation and Demonstration in Propositional and Functional Logic" and "The Propositional and Functional Logic of Derivation and Demonstration"[5]. They contain material which would later be incorporated in section 8: "Derivation and Demonstration", of [3]. He draws the distinction between demonstrational logic, exemplified by the systems of Russell-Whitehead, Hilbert-Ackermann and Hilbert-Bernays, and derivational logic, to which only Gentzen has come close with his system of natural deduction. In these drafts Popper formulates an idea much more radically than in his published articles: the logic of derivation should be primary and the logic of demonstration should be introduced via a definition of demonstrability as a second step. As indicated in the introduction, the reception of Popper's articles by the reviewers was rather negative. But not all reception was negative; William Kneale and Brouwer responded positively. Brouwer had presented three of Popper's articles to the Koninklijke Nederlandse Akademie van Wetenschappen and spoke very warmly about Popper's articles on logic. Even though Popper did not publish anything substantial on formal logic for the rest of his life, he continued to work on logical problems such as the quantum logic of von Neumann, the relation between non-classical negations and modality and, especially around 1950, on the different concepts of implication.

References

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[3] The article is in KPS 13,5; 14,8; 36,13. The title is proposed in the letter to Bernays from March 3rd, 1947.

[4] This definition of purity appears as definition D8.1 in [3], where it is discussed rather elliptically.

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