FROM COSMIC PATHS TO PSYCHIC CHAINS

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[Abstract]

Although I had been aware of Popper's work for well over 50 years, it is surprising – to me – that I had never thought of trying to reconstruct his relation to mathematics, and even more so, to its foundations. So I started by looking at what may have been his orientation to mathematical problems around 1920. Popper writes of this time: "At the University … I soon gave up going to lectures, with the exception of those in mathematics and theoretical physics." … "Only the Department of Mathematics offered really fascinating lectures. The professors of the time were Wirtinger, Furtwängler, and Hans Hahn. All three were creative mathematicians of world reputation … All these men … were demigods"[1]. [Freud had recorded a similar homage: if not directly towards mathematics, certainly towards what he took to be the methods of the sciences. The man that Freud had described – while he too was in in his twenties – as his "household God" was Hermann von Helmholtz.]

That's a good start, but what was Popper looking for? "I studied mathematics because … I thought that in mathematics I would learn something about standards of truth". "Standards of truth" rather than "standards of proof" already indicates a programme set within a somewhat wider domain than simply that of mathematics. [The text of his intellectual autobiography was written a half century after the experiences being described; but the text clearly has a high regard for historical accuracy]. In any case, at least we can say that a serious concern for the nature of mathematics was, in Popper's case, longstanding.

Already at this early period Popper had developed his distinction between critical thinking and dogmatic thinking, dogmatic theorizing representing "a stage that was needed if critical thinking was to be possible. Critical thinking must have before it something to criticize"[2]. If one were now to locate some of the earliest "dogmatic thinking" at – and before- the time of the Ionian Greeks, it might not be too hasty to attempt to reformulate this distinction as that between metaphysics and the progressive critical articulations of science[3].

Popper regularly comes back to give explanations – at a descriptive level – of his concern with "standards of truth". "Learn (such standards) from the way in which scientists and mathematicians proceed", giving special attention to "the mathematical background of physics": progress in science "constantly create(s)

[1] KRP: *Autobiography* [UQ pp39-40].
[2] [UQ p41].
[3] A reformulation that would also raise questions concerning a critical articulation of mathematics.

new theoretical and mathematical means for more nearly approaching" the truth[4]. As he describes his notion of metaphysical research programme, Popper makes the following comment: there have been "changes down the ages in our ideas of what a satisfactory explanation should be"[5]. This applies with special pertinence to questions of what can be called the metaphysics, or foundations, of mathematics. A central question here is whether or not formulations of mathematical problems constitute the leading ideas in the development of the philosophy of mathematics. In sketching the nature of this question, I will look at the proposals put forward by Arpad Szabó and by Popper as regards the relations of early Greek [Ionian and Eleatic] cosmology and mathematics.

Popper's relation to Szabó (as well as to the work of Imre Lakatos) itself needs some outlining. Popper's early distinction between deductive methodology in mathematics and deductive methodology in science needs much revision in the light of Lakatos's work on the centrality of a dialectic of refutation in the logic of mathematical discovery – on the importance, that is, of counter-examples to proof-claims in the progressive development of mathematics. I do not know when Popper first became aware of Szabó's work[6] – but this would raise a new perspective on the development of critical – or "hard hitting" - arguments in Eleatic philosophy, including problems in the philosophy of mathematics. These issues could be pursued piecemeal, but many of these questions can be put into a single perspective by introducing what Popper called his theory of transference.

Popper proposes a theory of transference – one that is central to his theory of science, and to his account of mathematics. In fact he proposes at least four such theories, and they stand in relation to each other in the form of a critical progression. This is an important method for Popper: "one of my principle methods of approach" [7]. He describes the method he proposes as follows: (1) "in logical problems" to "translate *all* (my italics) the subjective or psychological terms … into objective terms". This gives a means of transformation from Psyche to Logic. It should be noted that this involves a one-way relation. (2) Popper then extends this initial parallelism to a relation of transference between problems of scientific method and problems of logic. (3) He then augments this by a further extension to a transference between the history of science and logic[8]. But already there are some difficulties: they arise from the phrase "whenever logical problems are at stake", or rather from a combination of two phrases: "whenever logical problems are at stake … what is true in logic is true in scientific method

[4] [UQ pp 89 and 131, and N205].
[5] [UQ pp150-151].
[6] Szabó had published a series of papers starting in the 1950s; KRP would have been aware of IL's work by 1959.
[7] KRP: *Objective Knowledge*: [OK pp6-8].
[8] I have elsewhere described an identical form of transference between psychoanalysis and mathematics. The elements on the analytical side that require translation into mathematical structures are associative pathways (or logical thread, or signifying chains).

and in the history of science". The problem is that the "logic" of the first phrase, operating on the left hand side of the relation, ruins the transference claim[9].

Actually, Popper drops the first clause as he gives his version of his own *principle of transference*: "what is true in logic is true in psychology". And clearly this more succinct form – as well as avoiding the difficulty – requires a generalisation of his initial claim. He moves to such a generalisation: and it is a – further generalised – principle of transference. He calls it a "heuristic conjecture" – "quite generally, what holds in logic also holds … in psychology"; the omitted section here contains a new qualification: "provided it is properly transferred" – and after this conjecture, it is straightforward to propose the further transferences of form (2) and (3)[10]. Popper in fact extends these generalisations even further. This can be best seen in his [somewhat earlier] response to Szabó's presentation at the London Colloquium on the Philosophy of Science on 14th July 1965. Popper found Szabó's claim of the existence of transference relations between Euclidean geometry and Eleatic logic "very interesting". Szabó had been looking particularly at the question of the origins of axiomatisation, but again Popper generalises this problem-situation to that of the existence of transference relations between Euclid's geometry and Ionian and Eleatic cosmology. To assert the existence of a series of transference relations of this kind is a statement of the philosophy of mathematics: a statement within a programme – within a metaphysical programme – of mathematics[11]. So we have here a series of problems involving the philosophy of mathematics, and the relations between logic and the structure of the mind[12].

Any solution proposed to problems of the relation of "truth and proof" in this domain would hopefully provide an initial account of the autonomy of mathematics, allowing problems of the philosophy of mathematics to find their

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[9] At best the "all" subjective states becomes "some", and this by virtue of some – unspecified – logic. KRP was aware that there existed a wide class of logics, and that establishing a priority amongst them invoked a metaphysical series of claims about the problems of mathematics. He may not have been totally aware of the great multiplicity of contemporary logics, or of the variety of mathematical theories used in constructing very many of them.

[10] In proposing this version, KRP takes it that he is proposing a formulation that avoids "unconscious expectation" and "irrational" content. See [OK pp26 and 80].

[11] There is a need for the availability of certain functions to be able to be able to carry out such a programme. Its development would involve the introduction of a metaphysical programme for the theory of sets – including some assumptions as to the question of "strong" or "weak" logics [extending even into the theory of large cardinals].

[12] Clearly a central aim of these transference theses is to avoid any form of psychologism. I will sketch an account of how this is done using partial order relations – as developed for instance by Sierpinski and Garrett Birkhoff (whose texts Popper worked on over decades).

roots in mathematics, rather than proposing that a prior philosophy direct the orientation of the mathematics[13].

Popper has set out a transference between the psyche and logic, and given the nature of modern logic, this is effectively a transference relation between the psyche and mathematics. His proposal of course is for a one-way transference relation. The work of the Hungarian psychoanalyst Imre Hermann has extended this notion to a two-way transference relation between psychoanalytical structures and mathematics. I have elsewhere given accounts of the mathematics that would be involved in this, using in part the work of the Irish mathematician William Rowan Hamilton.

{Some of the themes involved here are: Transference in Dugald Stewart from 1811; in Freud from 1891, in Hermann from the 1920s onwards. This theme of transference – in its original philosophical and then psychoanalytical and then mathematical aspects – is at the centre of the problem-situation of the formalisation – that is, the mathematisation – of psychoanalysis. On these accounts, the problems of psychoanalysis can be solved by translating them into a corresponding mathematics, solving – where possible – the corresponding mathematical problems, and translating back [and vice-versa]. Any particular theory of psychoanalysis-mathematics transference can itself be tested by the ascertaining of clinical results. If such a transference is two-way then it is also at the centre of a reconstruction of the problem-situation of set theory from Zermelo onwards – of course with a pre-history starting in the initial years of the nineteenth century with Herbart}.

As for the title of this piece: it asserts a transference between the cosmological problems of the early Greeks, and problems formed by the pathways and chains in the mind. This presupposes that a clear structure is available for the spaces that constitute the psyche – and this has been generated by the mathematical philosophy of Herbart, which is the starting point of a programme that goes from Herbart to Riemann to Dedekind to Zermelo. *En route* it discovers the modern theory of topology, and builds set theory [the notion of chain – a technical notion of chain – is at the centre of Dedekind's set theory, as well as that of Zermelo].

[13] Szabó at times seems to deny this autonomy, finding directive principles for mathematics in the dialectic of Eleatic political philosophy. A contrary view is put forward by Kanamori [in his Appendix to *The Higher Infinite*], where he may well be referring to Szabó as he distinguishes the structure of mathematics from "the dialectical to and fro of philosophy". In terms of these relations, KRP – in his reply to Szabó – seems to give an autonomy to cosmology: of course, he is here referring to a cosmology that since the time of Thales and Anaximander has had mathematics embedded into it – or, to use KRP's terms, transferred into it.