

# Labour market recruiting with intermediaries\*

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## Abstract

We consider a Rothschild-Stiglitz-Spence labour market model and employ a centralised mechanism to coordinate the efficient matching of workers to firms. This mechanism can be thought of as operated by a recruitment agency, an employment office or head hunter. In a centralised descending-bid, multi-item procurement auction, workers submit wage-bids for each job and are assigned stable jobs as equilibrium outcome. We compare this outcome to independent, sequential hiring by firms and conclude that, in general, a stable assignment can only be implemented if firms coordinate to some extent. (JEL *C78, D44, E24, J41*. Keywords: *Matching, Multi-item auctions, Sequential auctions*.)

## 1 Introduction

“The entire recruitment market is estimated at more than \$300 billion a year. The market for executive search is approximately \$10 billion annually with middle-management recruitment estimated at more than \$30 billion a year on a global basis. The fragmented market includes many facets from MBA and college recruitment to career management, human resource outsourcing, candidate tracking and company job postings.” (Korn/Ferry Annual Report 2000.) Apart from private recruitment, there is the government employment office, industry matching programs such as the US National Resident Matching Program (and its international counterparts), central recruiting divisions in large corporations and the public sector. “Many large companies today spend in excess of \$30 million on search fees per annum and this is a growing phenomenon for sure.” (Scott A. Scanlon, CEO Hunt-Scanlon Advisors)

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The commercial intermediary `jobdumping.de` allows individuals to bid online for jobs. The individual asking for the lowest compensation is assigned the job. All non-negligible fees are paid by the firm offering jobs. These are given in percentages of the starting ‘reservation’-value of the job auction: for a value of €50,000, for instance, the fee paid to `jobdumping.de` is 1.6% (€800). We analyse precisely this type of labour market interaction through an intermediary.<sup>1</sup>

Why do firms use recruiters? We study this question as the problem of matching a number of heterogenous workers to a number of non-identical jobs in the well-known labour market model of Rothschild-Stiglitz-Spence.<sup>2</sup> If firms recruit through a single, centralised intermediary, this matching problem is similar to that of auctioning multiple items of non-identical goods to a number of heterogenous buyers demanding only a single good each. Demange, Gale, and Sotomayor (1986)—referred to as DGS below—solve this problem by devising an incentive compatible, ascending price mechanism for general preferences. We adopt this mechanism to our special setting where workers face some privately known cost of doing some job which is offset by the wage that job pays. We let workers bid the wages for which they are willing to do a particular job in a descending version of the DGS mechanism. Hence, the idea is that competition between workers drives down the wages paid by firms. By using a recruiter who reports the workers’ revealed types back to the firms, the latter can benefit from this competition and extract the highest possible surplus.

We contrast this centralised mechanism with a sequence of independent second-price sealed-bid wage auctions, each conducted separately by a single firm.<sup>3</sup> We show that only a very particular sequence of independent second-price auctions is able to implement the stable and efficient DGS assignment. For this sequence to prevail, firms require some coordination and information sharing among themselves which we ascribe to an intermediary.

## Related literature

Shapley and Shubik (1972) initiated the analysis of the assignment problem. Its application to the problem of assigning jobs to workers is studied in a series of classical papers by Crawford and Knoer (1981), Kelso and Crawford (1982), and Leonard (1983). As pointed out above, Demange and Gale (1985) and Demange, Gale, and Sotomayor (1986) are central to our analysis. Sequential auctioning of two jobs with single-crossing preferences is discussed by Elmaghraby (2003) and in a more general fashion by Kittsteiner, Nikutta, and Winter (2004) who also survey the relevant literature in detail.

The following section defines the model, section 3 states and discusses our results and section 4 illustrates these results by example. All proofs and details are presented in the appendix.

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<sup>1</sup> We are grateful to an anonymous referee for suggesting this direct application of our model.

<sup>2</sup> This model is described in detail, for example, in Mas-Colell, Whinston, and Green (1995, 13.D).

<sup>3</sup> By the revelation principle, the precise type of auction mechanism used is unimportant.

## 2 The model

There is a set  $\mathcal{N}$  of  $N$  workers competing for a set of vacant jobs  $\mathcal{M}$  containing  $M$  jobs. Each worker can accept at most one job. Since we are interested in the effects of competition among workers for jobs we assume that  $N \geq M \geq 2$ . Each worker—typically indexed by  $i \in \mathcal{N}$ —has a private skill type  $\theta_i$  which is ex-ante drawn from some known distribution over a subset of  $\mathbb{R}_+$ . Jobs—typically indexed by  $j \in \mathcal{M}$ —are ordered in increasing complexity (‘task level’)  $T^j \in \mathbb{R}_+$  as  $T^1, T^2, \dots, T^M$ . These jobs are exogenously given and commonly known.

Workers are equipped with quasi-linear preferences over wages  $w$  and private cost of effort  $c(\theta, T)$  summarised by

$$u_i(\theta_i, w^j, T^j) = w^j - c(\theta_i, T^j) \quad (1)$$

pinned down by the costless, increasing outside option  $w^o(\theta_i)$  at  $c(\theta, T = 0) = 0$ . The vector of all workers’ utilities is  $\mathbf{u}$ . We denote the set of all players’ outside options by  $\mathcal{O}$ . Outside options are not in  $\mathcal{M}$ —elements of which we call ‘inside options’—and workers matched to their outside option are called unmatched. The cost function is common knowledge and the same for all workers. For ease of exposition, we assume that, for all  $T$  and  $\theta$ , workers’ preferences satisfy the Spence-Mirrlees (‘single-crossing’) condition on the non-negative real numbers<sup>4</sup>

$$c_T(\theta, T) > 0, c_{TT}(\theta, T) > 0, c_\theta(\theta, T) < 0, c_{T,\theta}(\theta, T) < 0. \quad (2)$$

Thus worker  $i$  has an ‘indifference’ wage  $\tilde{w}_i^j$  for a job  $j$  such that  $\tilde{w}_i^j - c(\theta_i, T^j) = w^o(\theta_i)$  and worker  $i$  is indifferent between her outside option and job  $j$ . Therefore  $\tilde{w}_i^j$  is the lowest wage at which worker  $i$  is prepared to do job  $j$ . We denote the rent  $i$  obtains from job  $j$  at wage  $w^j$  by  $r_i^j = w^j - \tilde{w}_i^j$ . The set of workers demanding a particular job  $j \in \mathcal{M}$  at  $\mathbf{w}$  and worker  $i$ ’s demand set for jobs at wages  $\mathbf{w}$  are written, respectively, for  $r_i^j = w^j - c(\theta_i, T^j) - w^o(\theta_i)$ , as

$$\begin{aligned} D^j(\mathbf{w}) &= \{i \in \mathcal{N} : j \in \arg \max_{k \in \mathcal{M} \cup \mathcal{O}} \{w^k - c(\theta_i, T^k)\}\}, \\ D_i(\mathbf{w}) &= \{j \in \mathcal{M} \cup \mathcal{O} : j \in \arg \max_{k \in \mathcal{M} \cup \mathcal{O}} \{w^k - c(\theta_i, T^k)\}\}. \end{aligned} \quad (3)$$

There are  $M$  firms offering a single job  $j \in \mathcal{M}$  each. When the job is filled with *any* capable worker, the job generates a constant exogenous revenue of  $\bar{w}^j$ .<sup>5</sup> Firms’ preferences are given by  $v^j(\bar{w}^j, w^j) = \bar{w}^j - w^j$  and thus  $\bar{w}^j$  is the highest wage a firm is willing to pay to a worker accepting job  $j$ . Again the vector of all firms’ utilities is  $\mathbf{v}$ . We assume the vector of reserve wages  $\bar{\mathbf{w}}$  to be publicly known.

<sup>4</sup> None of our main results relies on this assumption. It is, however, typically required in labour market models.

<sup>5</sup> This implies that firms offer jobs for which they do not care about the hired worker’s type as long as this worker can fulfill her job. They decide only on the basis of cost minimisation. As firms are non-strategic about the type of worker they hire, alternative interpretations of our model are the market for delegation, outsourcing with centralised negotiation or centralised (inter-department) procurement.

The following is an adoption of a procedure devised by DGS to descending wage-prices:

- $t=1$ : The intermediary announces the starting wage vector  $\mathbf{w}(1) = (\bar{w}^1, \bar{w}^2, \dots, \bar{w}^M)$ . Each worker bids by revealing her demand  $D_i(\mathbf{w}(1))$  at wage  $\mathbf{w}(1)$ .  $D_i(\mathbf{w})$  is nonempty because if a worker accepts no job in  $\mathcal{M}$  and is unmatched, she still demands her outside option.
- $t+1$ : After bids are announced, if it is possible to assign each worker  $i$  to a job in her demand set  $D_i(\mathbf{w}(t))$  at price  $\mathbf{w}(t)$ , the procedure stops. If no such assignment exists, there must be some *overdemanded* set, that is, a set of jobs such that the number of workers demanding only jobs in this set is greater than the number of jobs in this set. The intermediary chooses a *minimal* overdemanded set, that is, an overdemanded set  $S$  such that no strict subset of  $S$  is an overdemanded set and *decreases* the offered wage for each job in this set by one unit. All other wages remain at the level  $\mathbf{w}(t)$ . This defines  $\mathbf{w}(t+1)$ .<sup>6</sup>

We follow DGS in assuming that all wages and workers' rents take integer values. This can be relaxed if a more involved mechanism were to be employed. Finally, the following definitions are standard:

1. A wage-vector  $\mathbf{w}$  is called *feasible* if, for all  $j \in \mathcal{M}$ , it is true that  $w^j \leq \bar{w}^j$ .
2. A feasible wage-vector  $\mathbf{w}$  is called *competitive* if there is an assignment  $\mu : \mathcal{N} \rightarrow \mathcal{M}$  such that if  $\mu(i) = j$ , then  $j \in D_i(\mathbf{w})$  for all  $i \in \mathcal{N}$ . In this case,  $\mu$  is said to be *compatible* with  $\mathbf{w}$ .
3. A pair  $(\mathbf{w}, \mu)$  is called a *competitive equilibrium* if (i)  $\mathbf{w}$  is competitive, (ii)  $\mu$  is compatible with  $\mathbf{w}$ , and (iii)  $w^j = \bar{w}^j \forall j \notin \mu(\mathcal{N})$ .
4. A feasible outcome  $(\mathbf{u}, \mathbf{v})$  with assignment  $\mu(i) = j$  compatible with  $\mathbf{w}$  is called (pairwise) *stable* if, for all  $(i, j) \in \mathcal{N} \times \mathcal{M}$ ,
  - (a) there is no job  $g \neq j \in \mathcal{M}$ , such that  $u_i(\theta_i, w^g, T^g) > u_i(\theta_i, w^{\mu(i)}, T^{\mu(i)})$ , that is, provides worker  $i$  with higher utility than his matched job  $j$  while
  - (b) worker  $\mu(g) \neq i \in \mathcal{N}$  demands job  $j$  at  $w^{\mu(i)}$ , that is, provides firm  $j$  with at least the same utility  $v(\cdot)$  as when matched to  $i$ .

The idea is that for a stable matching of two workers and two firms, no worker demands the job she is not matched to at a wage lower than that specified by the stable matching for that job. If there are more workers or firms, the same condition is required for any pair of workers and firms.

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<sup>6</sup> This implicitly defines the minimal bidding decrease  $\varepsilon = 1$ . Ties are the measure zero events of more than one worker having the same type. We break these with equal probability in either way.

### 3 Results

The proofs of the first two propositions differ only slightly from the original arguments developed by DGS in that a decreasing-price version of their mechanism needs to be employed. Proposition 6 is a slight extension of Demange and Gale (1985) to the case of  $N = M$ . Our restriction to single-crossing preferences compared to DGS plays only a role in the interpretation as a Rothschild-Stiglitz-Spence labour market model (propositions 4 & 5). None of the other arguments relies on single-crossing.

**Proposition 1.** *Let  $\mathbf{w}$  be the wage-vector obtained from the DGS mechanism. Then  $\mathbf{w}$  is the maximum competitive wage.*

**Proposition 2.** *If  $\mathbf{w}$  is the maximum competitive wage obtained from the DGS mechanism, then there is an assignment  $\mu$  such that  $(\mathbf{w}, \mu)$  is an equilibrium and  $\mathbf{w}$  is a competitive equilibrium wage-vector.*

Following Demange (1982), Roth and Sotomayor (1990, p212f) identify the DGS mechanism as a generalised Vickrey-Clarke-Groves (VCG) mechanism. Holmström (1979) shows that the VCG mechanism is the unique direct reporting mechanism with dominant strategies, efficient outcomes, and zero payments by losing bidders. Thus, with independent bidder types, any other (centralised) auction design leading to efficiency must involve the same wage-payments as the DGS mechanism.

**Proposition 3.** *The outcome  $(\mathbf{u}, \mathbf{v})$  under the competitive equilibrium  $(\mathbf{w}, \mu)$  is stable.*

A restriction to single-crossing preferences allows to ensure additional structure on the equilibrium assignment resulting from propositions 1–3. This restriction, however, is made only for the two following propositions. All other results hold for general preferences.

**Proposition 4.** *Under single-crossing preferences (2), competitive equilibrium wages  $\mathbf{w}$  in the DGS assignment are increasing in  $T$ .*

**Proposition 5.** *Under single-crossing preferences (2), the skill types  $\theta$  of workers matched to jobs in the competitive equilibrium assignment are increasing in  $T$ .*

We now introduce the concept of an *autonomous* match as a job, the wage element of which is determined by an unmatched outsider. We proceed to identify such a job in the DGS assignment. Since the equilibrium wage assigned to an autonomous job by the DGS mechanism can be implemented using a single second-price sealed-bid auction, we know that the *final* auction in a sequence of second-price auctions can implement the autonomous DGS job settlement.

**Definition.** A job match  $(j, w^j)$  is called *autonomous* if its wage component  $w^j$  either equals firm  $j$ 's reservation wage  $\bar{w}^j$ , or the indifference wage  $\tilde{w}_i^j - \varepsilon$  of an unmatched worker  $i$ .

**Proposition 6.** *Every DGS competitive equilibrium assignment  $(\mathbf{w}, \mu)$  contains an autonomous match.*

We now compare the outcome of the centralised mechanism with a sequence of independent second-price auctions which firms employ to hire individually. The precise information structure of this sequence of independent second-price auctions is not important since we do not attempt to derive an equilibrium of this sequential auction. We merely derive a necessary condition for this sequence to result in the VCG allocation. All we require is that firms are unable to coordinate on a particular sequence (by, for instance, choosing positions simultaneously) and for that it suffices to assume that firms draw their positions in the sequence of individual auctions at random. For concreteness, we assume the following structure for the sequential mechanism: Firms hire through a sequence of independent second-price, private sealed-bid auctions. No information is made public after each round of hiring except for the identity of the worker winning the job auctioned at that stage.

**Proposition 7.** *In order to implement the DGS equilibrium with a sequential mechanism, it is necessary that an autonomous job is auctioned last in a sequence of independent, non-autonomous job auctions.*

The previous proposition presents a weak necessary condition for the sequential mechanism to result in the VCG allocation. However, the idea of an autonomous job has surprising power: There can be assignments containing more than one autonomous match; indeed there are assignments containing only autonomous matches. Hence, there are preference-profiles where many sequences of one-shot auctions can lead to a stable assignment. In general, however, a random sequence of one-shot second-price, sealed-bid auctions cannot ensure that an autonomous match is auctioned last. Therefore, some amount of cooperation among the firms—perhaps in the form of an intermediary—is required to implement a stable competitive equilibrium assignment.

To appreciate the strategic difference between the DGS and the sequential mechanisms, it suffices to recall that winners of previous rounds do not participate in the sequential mechanism while they do participate in the DGS mechanism. In the DGS mechanism, workers tentatively assigned to different jobs bid down to their indifference wage plus the rent obtained from their tentative inside option. Hence, there are (weakly) more potential players at each round of the DGS mechanism than in the sequential version. These additional players cannot make a difference to the wage outcome if they are not better qualified than the players also present in the sequential mechanism. But if they are more efficient than the competitors in the sequential mechanism, their presence must lower the negotiated wages. The example in the following section 4 illustrates this effect for the case of three workers competing for two jobs.

## 4 An illustrative example

Consider the following example where a single worker  $L$  is best suited for more than one job: There are two jobs 1,2 with  $T^1 < T^2$  and three workers  $L, M, H$  with  $\theta_L < \theta_M < \theta_H$  with preferences and reserve wages as drawn below.

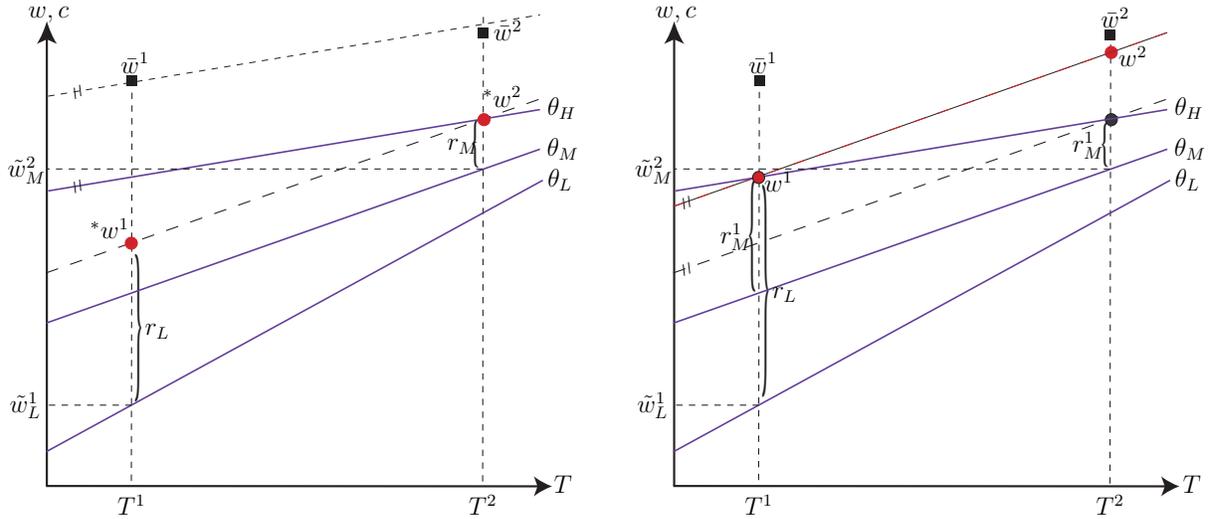


Figure 1: The stable DGS outcome (left). Job 2 is autonomous and thus auctioning job 2 first using independent sequential auctions is either not stable because  $r_M^1 > r_M^2$  or more expensive (right).

(1) The DGS mechanism results in the stable allocation of  $L$  to 1,  $M$  to 2 and the wage profile  $w^* = (\tilde{w}_M^1 + r_M - 2\varepsilon, \tilde{w}_H^2 - \varepsilon)$ . To see this, let the initial wage vector be  $w_0 = (\bar{w}^1, \bar{w}^2)$  resulting in the demand  $D_L(w_0) = D_M(w_0) = D_H(w_0) = \{1\}$ . Hence 1 is overdemand and the going wage  $w^1$  is decreased to some  $w_1 = (w_1^1, \bar{w}^2)$  at which  $H$  no longer demands 1 but prefers 2. Since, at that wage, 1 is still overdemand by  $L$  and  $M$ ,  $w^1$  is further decreased to some  $w_2 = (w_2^1, \bar{w}^2)$  at which  $D_L(w_2) = \{1\}$ ,  $D_M(w_2) = \{1, 2\}$  and  $D_H(w_2) = \{2\}$ . Hence both 1 and 2 are overdemand at  $w_2$  and both  $w^1, w^2$  are reduced simultaneously until  $w^2 < \tilde{w}_H^2$  at which point  $H$  switches demand to her outside option  $\mathcal{O}$  and  $D_L(w_3) = \{1\}$ ,  $D_M(w_3) = \{1, 2\}$  and  $D_H(w_3) = \{\mathcal{O}\}$ . Hence, at  $w^3$ , 1 is overdemand and a final  $\varepsilon$ -reduction in  $w^1$  results in each worker demanding a unique job (or outside option) at  $w^*$ . In this example, job 2 is autonomous since its wage is determined by  $H$ .

(2) Let us verify that the same outcome is reached when 1 is auctioned before the autonomous 2 using two independent second-price auctions: Start at some initial wage vector  $w_0 = (\bar{w}^1, \mathbb{E}[w^2 | N^2 = 1, \theta] = \bar{w}^2)$  resulting in the demand  $D_L(w_0) = D_M(w_0) = D_H(w_0) = \{1\}$ . As before, 1 is overdemand and the going wage  $w^1$ —which is the only wage which can be adjusted sequentially—is decreased to some  $w_1 = (w_1^1, \mathbb{E}[w^2 | N^2 = 1, \theta] = \bar{w}^2)$  at which  $H$  no longer demands 1 but quits the auction for job 1. If  $H$  does not misrepresent her preferences, this reveals her true type to her opponents who conclude that if they quit next, the should

expect wages of  $\mathbb{E}[w^2|N^2 = 2, L] \leq \tilde{w}_H^2$  and  $\mathbb{E}[w^2|N^2 = 2, M] \leq \tilde{w}_H^2$  for the next-auctioned job 2. At the same time, 1 is still overdemanded by  $L$  and  $M$ . Further reducing  $w^1$  to some  $w_2 = (w_2^1, \mathbb{E}[w^2|N^2 = 2])$  must result at some point in  $D_L(w_2) = \{1\}, D_M(w_2) = D_H(w_2) = \{\emptyset\}$  which clears the market for 1 at the same wage  ${}^*w^1 = \tilde{w}_M^1 + r_M - 2\varepsilon$  as above. The single subsequent second-price auction at 2 terminates at  ${}^*w^2 = \tilde{w}_M^2 - \varepsilon$ . Since this is the VCG outcome, no worker has incentives to misrepresent her true preferences.

(3) The same is not true if the autonomous job 2 is independently auctioned before 1: Since 1 is auctioned last and we want to implement a stable allocation, we know that  $M$  must be assigned to 2 leaving only  $L$  and  $H$  in the contest for 1. Thus a second-price auction must result in  $w^1 = \tilde{w}_H^1 - \varepsilon > {}^*w^1$ . At this wage, however,  $M$  prefers 1 to 2, thus 1 is overdemanded, and in order to reach a stable outcome, the sequential mechanism must offer a wage  $w^2 > {}^*w^2$  to  $M$ . Hence in this example, decentrally negotiated stable wages exceed the wages negotiated through the recruiter.

(4) If, alternatively, the workers' preferences are such that both offered jobs are autonomous, then any sequence of independent second-price auctions will implement the stable outcome.

## Conclusion

We show that the stable and efficient assignment obtained from a centralised labour market cannot be in general implemented through a sequence of independent auctions. This result is due to a centralised intermediary's informational advantage over the individual firms embodied in his knowledge of all workers' bids on all jobs.

## Appendix

The proofs of propositions 1 and 2 follow directly from Demange, Gale, and Sotomayor (1986). Their proofs are therefore omitted.

*Proof of proposition 3.* Since  $\mathbf{w}$  is competitive, if  $\mu(i) = j$ , then  $j \in D_i(\mathbf{w})$ . Hence in equilibrium every worker gets the job which gives her the highest rent at  $\mathbf{w}$  and nobody envies somebody else's job not in her demand set. Moreover, since our mechanism only terminates when there are no overdemanded jobs remaining, there is no worker  $h \neq i$  who prefers to do job  $j = \mu(i)$  for  $\hat{w}^j < w^j$  over any job in  $D_h(\mathbf{w})$ .  $\square$

*Proof of proposition 4.* Suppose the opposite is true and consider any two matched jobs  $1, 2 \in \mathcal{M}$  with  $T^1 < T^2$  and  $\tilde{w}^1 \geq w^2$ . Then 1 must be overdemanded from single-crossing of preferences contradicting the rules of the DGS mechanism.  $\square$

Proof of proposition 5. Suppose the opposite is true and consider any two jobs  $1, 2 \in \mathcal{M}$  with  $T^1 < T^2$  matched to two workers  $L, H \in \mathcal{N}$  with  $\theta_L < \theta_H$  such that  $H$  is assigned to job 1 and  $L$  is assigned to job 2. From the previous proposition we know that  $w^1 < w^2$ . But then, from single-crossing, player  $L$  must prefer job 1 to his equilibrium assignment and is willing to do it for less than  $w^1$ . Hence job 1 is overdemanding contradicting the rules of the DGS mechanism.  $\square$

Proof of proposition 6. This argument extends Demange and Gale (1985) to the case of  $N = M$ . Since the DGS mechanism's equilibrium assignment is stable, we have each matched worker  $i = D^j$ 's indifference curve passing through his assigned job  $j = \mu(i)$  at the point  $(T^j, w^j)$ . On the one hand, if a matched job  $j$  was never overdemanding, then its matched wage is  $\bar{w}^j$  because  $i = D^j$  is the only worker who ever demanded  $j$ . If, on the other hand, a job was overdemanding at some stage prior to the equilibrium assignment, then there is an intersection of the indifference curves of the equilibrium match for  $j$  and the worker  $h \neq i$  who switched his demand from  $j$  at  $w^j + \varepsilon$ . If worker  $h$  is unmatched by  $\mu$ , then  $j$  is autonomous and  $w^j + \varepsilon = \tilde{w}_h^j$ . If there is a match  $k = \mu(h)$ , then  $j$  is non-autonomous. Thus each non-autonomous job  $j$  is connected through the unique indifference curve of worker  $D^k$  passing through  $(T^j, w^j + \varepsilon)$  to another assigned job  $k$ . The demand withdrawal of worker  $D^k$  from  $j$  to  $k$  decides the wage  $w^j$ . As the wage on each non-autonomous job is determined by exactly one matched worker, we need at least one unmatched worker deciding the equilibrium wage on some job if  $N \geq M$ . Hence there must be at least one autonomous job in any DGS assignment.  $\square$

Proof of proposition 7. In a single second-price, sealed-bid auction, it is well known from Vickrey (1961) to be a weakly dominant strategy to bid one's true valuation. Thus in order to duplicate the assignment of the DGS mechanism, it is necessary to auction an autonomous job last in any sequence of second-price, sealed-bid auctions.  $\square$

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