Labelling contests with endogenous precision*

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Abstract

This paper introduces a novel type of imperfectly discriminatory contest which endogenises the cost of compiling the relative ranking it is based on. As a first application, we propose a simple theory of labelling for credence or experience goods of differing quality. We model the competition for the first, second, etc. labels as a rank order tournament in which firms can jointly control the ranking precision through the release of individual information. This information may be interpreted as endogenously established (input for) labelling agencies, experts or regulatory bodies. While the labels can be seen as a public good guiding the consumers' purchasing decisions, individual firms have incentives to free ride on the competitors' information emission. The theory seems to be applicable to many industries including advertising, investment rating, the production (and pirating) of computer software, movies or music, etc. (JEL *C7, D7, H4, M3.* Keywords: *Labelling, Contests, Advertising.*)

1 Introduction

Typically, contests and tournaments are modelled such that some 'black box' technology ranks the contestants' efforts or expenditures. The precise properties of this ranking technology such as, for instance, the precision of the ranking are usually kept exogenous. This paper introduces a new type of contest which allows contestants' to strategically control the precision of the employed ranking technology in the form of costly outlays made by the players. It is intuitive that the collection of imprecise statistics is viewed as less costly than the assembly of the perfect ranking embodied in, for instance, the all-pay auction. In this spirit, we view the fully discriminatory all-pay auction as the limiting case of costlessly observable efforts while our model endogenously defines a contest's ranking precision by the cost of obtaining noisy statistics on efforts.

As a first application of this theory, we use a tournament to provide a ranking of vertically differentiated goods or services with a credence or experience aspect to the uninformed public.[1]

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[1] "A *credence good* is a good (or service) whose utility is difficult or impossible to ascertain for the consumer. In contrast to *experience goods*, this utility is difficult to gauge even after consumption." (Wikipedia)

In the applications we have in mind, different qualities of some good cannot be ranked by the consumers. An ordinal ranking of these goods—created, for instance, by experts on the basis of information provided by the producers of the goods—is, however, perfectly useful to inform the public's consumption decisions. Applications of our theory include the labelling of organic produce (Soil Association, Demeter, Which?), medical drugs (General Practitioners, Food and Drug Administration approval), general practitioners and doctors (Medicare, NHS), academic departments (research evaluations), novels (Pulitzer, Man Booker Prize), politicians and cars (media), software, games and movies (critics), financial products and firms (rating agencies), or political issues of conscience (activists).

The industry we use as the leading example to motivate our setup is the marketing and distribution of computer software and movies, e-books, or music in digital form. These clearly have a credence or experience aspect: one needs to read a book or watch a movie to know its quality. Likewise, one needs substantial expertise to be able to evaluate software. The deployment discussion for Microsoft's Windows 7, for instance, is extensive and wholly impenetrable to the uninitiated.[2] Moreover, even after mastering one product, one still has no means of comparison with other products.

For the movie industry, rankings are provided, among others, by the Internet Movie Database `imdb.com` or VideoCD quality `vcdq.com`. The roles of these web sites are comparable to film review magazines. Another type of (commented) ranking combined with download statistics is provided through piracy sites such as The Pirate Bay (`thepiratebay.org`) or Demonoid (`demonoid.me`).[3] Posting (links to) stolen software for download creates a loss in potential profits for the producers. But the statistics generated by the (mostly illegal) downloads also create an informational externality which makes a choice based on other users' perceived qualities possible. There is, therefore, a social benefit to the activities of the above mentioned facilitators of illegal downloading since they provide a public service instrumental for informed choice. This service is genuine—since the pirates are seen to violate the commercial interests of the ranked copyright owners, their service is credible.[4] The producers themselves cannot credibly provide relative rankings and there exist no comparably unbiased alternative rankings which are not to some degree captured by producers.

The contribution of this paper is twofold. On the theoretical side we provide the first analysis of contests with endogenously chosen ranking precision.[5] This makes applications to (partial) credence or experience goods markets possible where, at some cost, complex product descriptions and

[2] See, for example, `http://technet.microsoft.com/en-us/deployment/default.aspx`.

[3] The rankings are real numbers in $[0, 10]$ at `imdb.com` and the usual $\star$ system at `vcdq.com`. The Pirate Bay and Demonoid are public BitTorrent trackers providing detailed user comments on the items they make available for download. What matters most for evaluation purposes are the number of downloads ('seeders' and 'leechers'). The Pirate Bay has more than 5 million registered users and is "one of the world's largest facilitators of illegal downloading" and "the most visible member of a burgeoning international anti-copyright or pro-piracy movement" (Los Angeles Times, 29-Apr-07). According to its web site, The Pirate Bay indexed 3,578.781 torrents and served "about 16.888,498.602,639.360 bytes of multi media action" on its eighth birthday (15-Sep-2011).

[4] There are examples of the copyright owners using these rankings strategically. See, for instance, `http://www.wired.com/threatlevel/tag/nude-nuns-with-big-guns/`.

[5] There are alternative interpretations of our model in which the endogenous parameters of our ranking technologies are interpreted as, for instance, the scale returns of the investments made. For references to such alternative interpretations see, among others, Nti (1999).

evaluations are translated into a simple ordinal ranking. Secondly, in these stylised applications, we integrate both the consumer and producer side in a novel way such that consumer demand reacts endogenously to the firms' choice of qualities and information dissemination policies.

1.1 Related literature

The idea that in many circumstances rank order tournaments can be employed to achieve socially beneficial outcomes is due to Lazear and Rosen (1981). The idea has found numerous applications and extensions, for instance in the work of Nalebuff and Stiglitz (1983), Dixit (1987), Moldovanu and Sela (2001), or Siegel (2009). For a detailed survey of the contests literature see the comprehensive Konrad (2008). To our knowledge, however, there is no prior contribution which allows contestants to endogenously control the precision of the employed ranking technology in a strategic fashion. To that end, we model a contest's ranking precision as endogenously arising from costly outlays made by the ranked players. This seems plausible as the measurements literature, as surveyed for instance by Kurshid and Sahai (1993), typically argues that ordinal statistics are inherently cheaper to produce than cardinal statistics. By extension of this argument, one may view the collection of imprecise statistics as less costly than assembling the perfect ranking technology embodied in, for instance, the all-pay auction. In this sense, the fully discriminatory all-pay auction case can be thought of as the limiting case of costlessly observable efforts (or payments) while the non-fully discriminatory branch of the literature limits a contest's assignment precision by the cost of obtaining noisy statistics of efforts. The present paper provides the first model in which this cost is made explicit and used by the players as the basis for endogenously choosing a contest success function's precision.

Research on credence goods was initiated by Darby and Karni (1973) and found path breaking applications in, for instance, Pitchik and Schotter (1987), Taylor (1995), Emons (1997), Feddersen and Gilligan (2001), Pesendorfer and Wolinsky (2005), Fong (2005), and Dulleck and Kerschbamer (2009). Extensive experimental evidence on the determinants for efficiency in credence goods markets has been collected by Dulleck, Kerschbamer, and Sutter (2011). The case of experience goods is discussed, among others, by Nelson (1970) and Milgrom and Roberts (1986) with an intermediate attempt by Hahn (2004). Recent interest has been spurred by applications to competition policy, health care and the regulation of legal counselling. We hope that the recent and comprehensive survey presented by Dulleck and Kerschbamer (2006) allows us to keep our review of this literature minimal. At any rate, we are unaware of a previous application of contest-driven demand to the analysis of credence or experience markets.

The literature on labelling is well developed.[6] See, for instance, Lerner and Tirole (2006), Baksi and Bose (2007), Roe and Sheldon (2007), Lerner, Farhi, and Tirole (2010) or Harbaugh, Maxwell, and Roussillon (2011) and the papers cited therein. As far as we can tell, our contest setup is a novel approach to the labelling problem and there seem to be no directly applicable papers in this field. Barigozzi, Garella, and Peitz (2009) employ a contest to develop a (comparative) advertising

[6] Ashforth and Humphrey (1997) define *labelling contests* as the (unrelated) situation where "two or more stakeholders attempt to define divergent realities for a given audience" and motivate their definition with examples from sports.

model. They do not, however, consider ranking precisions, credence goods or model the demand side. In general, and confirmed by a recent review by Bagwell (2007), the advertising literature does not seem to have developed the idea of contests in the direction of our analysis.

2 The model

2.1 Supply side

There are two risk-neutral firms $\mathcal{N} = \{1, 2\}$ each of whom produces a good of quality θ_i, $i \in \mathcal{N}$ distributed according to $\theta_i \sim F_{[0,\bar{\theta}]}$, $\bar{\theta} \in \mathbb{R}_{++}$, with positive density $f(\theta_i)$.[7] Throughout the analysis we only make use of the ratio of these qualities $x_i = \theta_i/\theta_j$, $i = \{1, 2\}$ and $j = 3 - i$, and write $x = \theta_1/\theta_2$ if there is no danger of confusion.[8] Without loss of generality we re-index firms such that $\theta_1 \geq \theta_2$. We assume that these qualities are mutually known among firms but that only the distribution of qualities F is known to the consumers. In the basic model, production and distribution of the goods are assumed to be costless. Once the good is produced, there is nothing a firm can do to alter its quality.[9]

Firm i, however, can choose to release information $\varepsilon_i \in \mathbb{R}$ on its product. Together with the other firm's emitted information ε_j, this information determines the precision that is used (for instance by some labelling agency) to rank the products. In particular, we assume that the ranking precision is determined by the sum of available information $r = \varepsilon_1 + \varepsilon_2$. This information r also affects the level of product differentiation in the consumer market and thus the firm's expected profit represented by the winner's and loser's prizes $P^1(r)$, $P^2(r)$ in the contest. For the parts of the analysis where we deal with exogenous prizes, we assume that $P^1(r) > P^2(r) \geq 0$ for $r > 0$. More precisely, we assume that firm $i \in \mathcal{N}$ maximises

$$\max_{\varepsilon_i} u_i(\theta, \varepsilon) = q(x_i, r)P^1(r) + (1 - q(x_i, r))P^2(r) - c(|\varepsilon_i|) \tag{1}$$

where the information cost $c(|\varepsilon_i|)$ is strictly convex with $c(0) = 0$ and $q(x_i, r)$ is player i's probability of being ranked first. We write θ and ε for the full vectors (θ_1, θ_2), and $(\varepsilon_1, \varepsilon_2)$, respectively. We assume that this noisy ranking of the firms' qualities $q(\theta, \varepsilon)$ is both observable and verifiable, that $q_i(\cdot)$ is strictly increasing in θ_i, strictly decreasing in θ_j, equal to $^1/_2$ for identical qualities, twice continuously differentiable, and zero for $\theta_i = 0$ with $\theta_{j \neq i} > 0$, $j = 3 - i$. Moreover, we assume that $q_i(\cdot)$ is strictly increasing in $r = \varepsilon_i + \varepsilon_j$ if $\theta_i > \theta_j$.

[7] Although we generalise some of our results in later sections of this paper, we lay out the model in the main body of the paper for two firms only. With respect to the obtained intuition, this is without loss of generality.
[8] In case of the probability zero event that $\theta_j = 0$, we set $x_i = \infty$.
[9] We extend the model to allow for the sequential or simultaneous choice of quality and precision in section 5.

2.2 Demand side

Consumers are represented through a distribution of preferences $\mu \sim G_{[0,s]}$, $s \in \mathbb{N}$, with continuous and strictly positive density $g(\mu) > 0$.[10] Apart from these private preferences, the main element informing individual demand is a ranking of the qualities θ_1 and θ_2 arising spontaneously following the firms' release of information ε.[11] In the absence of a ranking and because we are in a credence market, we assume that consumers cannot distinguish between products. Products of identical expected qualities are assumed to be sold at the same price with the firms sharing expected profits equally. Consumers do not know the realisation of product qualities but can form expectations of these based on the commonly known distribution F. Consumers can observe the total amount of information in the market (i.e., the ranking precision) r, but not the individually emitted components ε_i. The utility of a type-μ consumer is assumed to be quasi-linear with

$$v(\mu, \theta) = \mu\tilde{\theta} - \tilde{p} \geq 0 \tag{2}$$

where $\tilde{p}$ is the price paid for a product of quality $\tilde{\theta}$.

In this setup, consumer demand only depends on expectations, that is, on the order statistics $f_{(k:n)}(\hat{\theta})$ giving the probabilities of the random variable $\hat{\theta}$ coming k^{th}, $1 \leq k \leq n$, among $n = 2$ independent random draws from the distribution of qualities $F(\hat{\theta})$. We denote the expectation of the k-ranked product quality by

$$\mathbb{E}[\Theta_{(k:n)}] = \int_0^{\bar{\theta}} \theta f_{(k:n)}(\hat{\theta}) d\hat{\theta}. \tag{3}$$

In our labelling contest, a firm's prize for coming k^{th} is the expected consumer demand captured by the k^{th}-labelled product given the observed ranking q. To determine this demand, we use a vertical product differentiation model where the expected quality is signalled through the product rank.[12] Given their mutually known qualities, firms decide on both their optimal, rank-dependent prices and the amount of information to release. Thus, the game consists of two stages, first firm $i = 1, 2$ chooses and announces prices (p_i^1, p_i^2) conditional on each possible rank k.[13] This defines the prizes P^1, and P^2 for the ranking tournament in which the firms subsequently choose ε_i.

It is useful to define the consumers' expectation of the k^{th}-ranked product quality for an observed

[10] We only use s as a scaling parameter in order to get 'nice' numerical example values.

[11] This strategically emitted information may be interpreted as the number of endogenously generated experts performing a ranking of products. Under this interpretation, we model non-strategic experts using some technology (embodied in the contest success function) to give advice to consumers in a credence market. A stochastic rationale for the use of success functions is given, among others, by Jia (2008) and Jia, Skaperdas, and Vaidya (2011).

[12] The classic references are Gabszewicz, Shaked, Sutton, and Thisse (1981) and Shaked and Sutton (1983).

[13] Since consumers' valuation only depend on the ranking of the firms—everything else being uninformative—each firm faces the same optimisation problem for choosing p_i^k and thus selects the same equilibrium vector of conditional product prices. Therefore, a timing under which firms decide ex-post on prices yields the same result. It is easy to see that this is the only equilibrium: since consumers have no credible information and firms no unique way of differentiation, each differential prize—if credibly signalling quality—would be matched by the opponent.

ranking. For rank $1 \leq k \leq n = 2$, this expectation is denoted by

$$\Lambda^k = \sum_{k=1}^{n} \tilde{q}_i^k(\tilde{x}, r)\, \mathbb{E}[\Theta_{(i:n)}] \tag{4}$$

where we denote the consumers' rank-dependent winning expectations over q by

$$\tilde{q}^1 = (\tilde{q}_1^1(\tilde{x}, r), \tilde{q}_2^1(\tilde{x}, r)) = \left(\frac{1}{1+\tilde{x}^{-r}}, \frac{1}{1+\tilde{x}^{r}}\right), \text{ and } \tilde{x} = \frac{\mathbb{E}[\Theta_{(1:2)}]}{\mathbb{E}[\Theta_{(2:2)}]}. \tag{5}$$

(The rank-dependent expectations of losing $\tilde{q}^2$ are defined accordingly.) Given an observed ranking q and announced prices $p = \left((p_i^1, p_i^2), (p_j^1, p_j^2)\right)$, a marginal consumer of valuation $\hat{\mu}_2^1$ is indifferent between buying the 1st- and 2$^{\text{nd}}$-ranked products iff

$$\mu\Lambda^1 - p_1 = \mu\Lambda^2 - p_2 \tag{6}$$

resulting in the vector of cutoffs

$$\hat{\mu} = \left(\hat{\mu}_1^0 = s,\ \hat{\mu}_2^1 = \frac{p_1 - p_2}{\Lambda^1 - \Lambda^2},\ \hat{\mu}_3^2 = \frac{p_2}{\Lambda^2}\right). \tag{7}$$

Given these cutoffs, the first and second ranked firms maximise their profits by choosing p_1^* and p_2^*, respectively, such as to

$$\begin{aligned} \arg\max_{p_1} P^1(r) &= p_1 \int_{\hat{\mu}_2^1(r)}^{\hat{\mu}_1^0 = s} g(\mu) d\mu = p_1 \left(G(\hat{\mu}_1^0) - G(\hat{\mu}_2^1)\right), \\ \arg\max_{p_2} P^2(r) &= p_2 \int_{\hat{\mu}_3^2(r)}^{\hat{\mu}_2^1(r)} g(\mu) d\mu = p_2 \left(G(\hat{\mu}_2^1) - G(\hat{\mu}_3^2)\right). \end{aligned} \tag{8}$$

As usual, we write the vector of prizes $(P^1(r), P^2(r))$ as P. Both $P(r)$ and $p(r)$ are used in the firms' maximisation problem of choosing ε.

Proposition 1. *For any number of firms $n = |\mathcal{N}|$ and any distribution G of consumer tastes μ with strictly positive and weakly concave density g satisfying the requirement that*

$$\frac{g(\mu)}{1 - G(\mu)} \text{ is strictly increasing,} \tag{9}$$

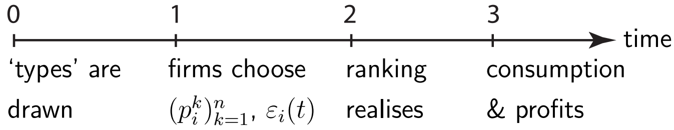
there exists an equilibrium vector of announced prices $p_1 < p_2 < \cdots < p_n$.[14]

2.3 Timing and information

Both prizes P and prices p are functions of the available information, i.e., the ranking precision $r = \varepsilon_1 + \varepsilon_2$. We are looking for (subgame perfect) asymmetric, pure strategy equilibria where each

[14] The proof is derived from Gabszewicz, Shaked, Sutton, and Thisse (1981). Since their working paper seems to be hard to obtain and our setup is slightly different, we nevertheless state the full proof in the appendix.

firm chooses pairs $((p_i^k)_{k=1}^2,\ \varepsilon_i)_{i=1}^2$. The complete timing of the interaction is shown below.



Consumers cannot observe the firms' individual ε, but they can observe the amount of overall available information r. Consumers use this information in determining the precision with which the ranking of product labels is correct, i.e. corresponding to the true order of qualities. The yardstick we use to measure the effects of labelling is the initial, unlabelled credence market. After an illustrative example we discuss the formal structure of the game giving rise to an endogenous ranking precision and characterise the firms' equilibrium behaviour. We extend the model in section 5 to allow for the sequential or simultaneous choice of quality and precision and present results under both symmetric and asymmetric market structures. We finish with an efficiency and welfare analysis and discuss whether the introduction of labelling is socially beneficial. All generalisations, proofs and details can be found in the appendix.

3 Example

Consider the following simple example with two firms of uniform quality and uniform preferences $\theta, \mu \sim U_{[0,\bar{\theta}=s]}$. Qualities θ_i are unobservable to the consumers but known among competitors. Thus, to the consumer, the expected quality of a product is given conditional on its rank. We award a single label to the firm ranked first and a firm chooses i) the price of her product conditional on the product's rank and the distribution of consumer preferences and ii) the resources ε_i it wishes to expend on influencing the overall ranking precision and demand.

We start by modelling the demand side. Expected qualities are obtained through summing up the k^{th} order statistic among n independent draws (3) and denoted by $\mathbb{E}[\Theta_{(k:n)}]$. For the Uniform distribution on the s-scaled unit interval these expectations are just $\mathbb{E}[\Theta_{(k:n)}] = s\frac{n-k+1}{n+1}$, $n = 2$, $k = 1, 2$. Given an observed ranking, for $\tilde{x} = \mathbb{E}[\Theta_{(1:2)}]/\,\mathbb{E}[\Theta_{(2:2)}]$, the consumers assess the expected qualities of the k^{th}-ranked products as

$$\Lambda^1 = \frac{1}{1+\tilde{x}^{-r}}\,\mathbb{E}[\Theta_{(1:2)}] + \frac{1}{1+\tilde{x}^{r}}\,\mathbb{E}[\Theta_{(2:2)}],\ \Lambda^2 = \frac{1}{1+\tilde{x}^{r}}\,\mathbb{E}[\Theta_{(1:2)}] + \frac{1}{1+\tilde{x}^{-r}}\,\mathbb{E}[\Theta_{(2:2)}]. \tag{10}$$

A 'type'-μ consumer is indifferent between the first- and second ranked products iff $\mu\Lambda^1 - p_1 = \mu\Lambda^2 - p_2$, i.e., we obtain the market cutoffs[15]

$$\hat{\mu} = (\hat{\mu}_1^0 = s,\ \hat{\mu}_2^1 = \frac{p_1 - p_2}{\Lambda^1 - \Lambda^2} = \left(3 + \frac{6}{2^r - 1}\right)(p_1 - p_2),\ \hat{\mu}_3^2 = p_2/\Lambda^2 = \left(3 - \frac{3}{2 + 2^r}\right)p_2). \tag{11}$$

Given these cutoffs, the first- and second-ranked firms maximise her profit by choosing p_1 and p_2,

[15] Notice that, because $\hat{\mu}_2^3 > 0$, the market will not be fully served.

respectively through

$$\max_{p_1} P^1 = p_1 \int_{\hat{\mu}_2^1}^{\hat{\mu}_1^0} g(\mu) d\mu = p_1(\hat{\mu}_1^0 - \hat{\mu}_2^1) \text{ and } \max_{p_2} P^2 = p_2 \int_{\hat{\mu}_3^2}^{\hat{\mu}_2^1} g(\mu) d\mu = p_2(\hat{\mu}_2^1 - \hat{\mu}_3^2). \tag{12}$$

Maximisation wrt p_i and solving gives the optimally announced, rank-dependent prices as

$$p_1^*(r) = s^2 \frac{2\left(2^r - 1\right)\left(2^{r+1} + 1\right)}{3\left(2 + 9 \times 2^r + 7 \times 4^r\right)},\ p_2^*(r) = s^2 \frac{2^r + 4^r - 2}{3\left(2 + 9 \times 2^r + 7 \times 4^r\right)} \tag{13}$$

resulting in the equilibrium cutoffs

$$\hat{\mu}_1^0 = s,\ \hat{\mu}_2^1 = s\frac{3 \times 2^r}{2 + 7 \times 2^r},\ \hat{\mu}_3^2 = s\frac{2^r - 1}{2 + 7 \times 2^r} > 0 \tag{14}$$

giving, in turn, the rank dependent contest prizes as functions of the available information as

$$\begin{aligned} P^1(r) &= (\hat{\mu}_1^0 - \hat{\mu}_2^1)p_1^* = s^3 \frac{4\left(2^r - 1\right)\left(1 + 2^{r+1}\right)^2}{3\left(1 + 2^r\right)\left(2 + 7 \times 2^r\right)^2}, \\ P^2(r) &= (\hat{\mu}_2^1 - \hat{\mu}_3^2)p_2^* = s^3 \frac{\left(2^r - 1\right)\left(2 + 2^r\right)\left(1 + 2^{r+1}\right)}{3\left(1 + 2^r\right)\left(2 + 7 \times 2^r\right)^2}. \end{aligned} \tag{15}$$

How these prizes result from consumer demand is illustrated in figure 1.

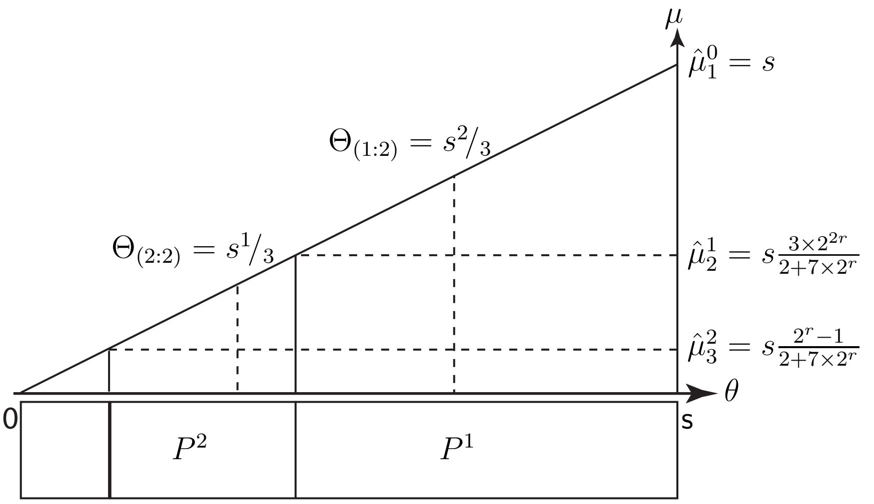


Figure 1: The labelled example market.

Given the prizes (15), we turn to the supply side and state firm $i = 1, 2$'s maximisation problem (under mutually known $x_i = \theta_i/\theta_j$) as

$$\max_{\varepsilon_i} \frac{1}{1 + x_i^{-r}} P^1(r) + \frac{1}{1 + x_i^r} P^2(r) - \frac{\varepsilon_i^2}{2}, \text{ where } r = \varepsilon_i + \varepsilon_j \text{ and } j = i - 3. \tag{16}$$

In order to get numerical values, we fix $\theta_1 = {}^3/_4$, $\theta_2 = {}^1/_4$ (i.e., $x = 3$), and $s = 4$. After taking derivatives wrt ε_i, we find the asymmetric equilibrium candidate as

$$\varepsilon_1^* = 1.847, \;\; \varepsilon_2^* = -0.088, \text{ implying that } r^* = 1.759.$$

Figure 2 verifies this candidate as pure strategy equilibrium.

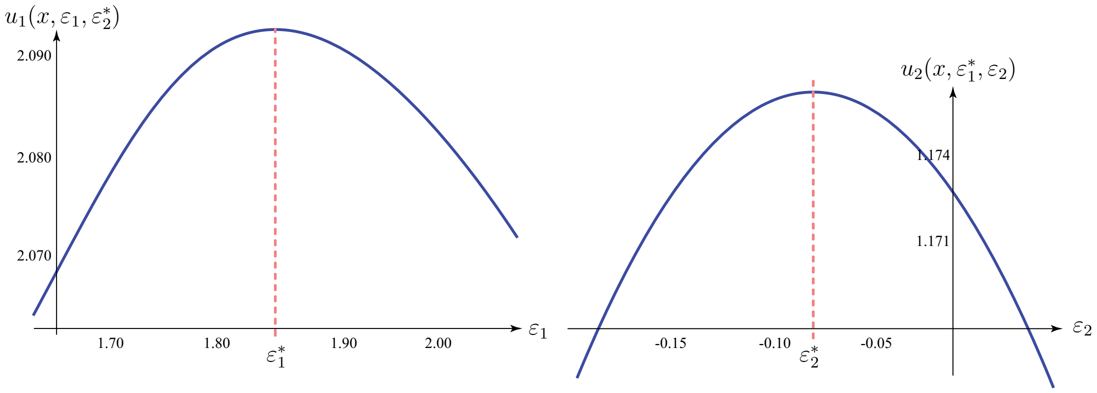


Figure 2: The two players' optimal choice of ε_i in asymmetric equilibrium.

Notice that the higher quality firm chooses to improve the ranking precision by supplying a positive amount of information while the lower quality firm obfuscates the ranking by decreasing the total ranking precision. The firms information policies, however, do not exactly cancel each other out and the resulting market precision is an informational improvement over the status quo of the unranked market $r = 0$. We will show in section 6 that, in this simple example, labelling a single product (and thus creating a full ranking) improves consumer welfare over the unlabelled case. The next section derives more general insights into the machinations of precision contests and derives the general analytic form of the firms' information dissemination policy functions.

4 Analysis

We begin the analysis by stating a simple but useful general property of precision contests.

Lemma 1. *Consider player $i \in \mathcal{N}$ with objective* (1)*. A necessary condition for a maximum is*

$$c'(|\varepsilon_1|) + c'(|\varepsilon_2|) = P'^1(r) + P'^2(r). \tag{17}$$

Equipped with this Lemma, we now develop our intuition step by step, starting with a simple case leading up to the full application we study. Before we analyse the contest, however, let us quickly remark that (17) cannot, in general, be socially optimal because a planner maximising benefits over costs (in the absence of a consumer population) solves the problem

$$\max_{\varepsilon_1, \varepsilon_2} q(x,r)\left(P^1(r) + P^2(r)\right) + (1 - q(x,r))\left(P^1(r) + P^2(r)\right) - c(|\varepsilon_1|) - c(|\varepsilon_2|) \tag{18}$$

which results in the foc

$$c'(|\varepsilon_1|) + c'(|\varepsilon_2|) = 2(P'^1(r) + P'^2(r)). \tag{19}$$

Thus, the precision of the privately provided product ranking is necessarily socially inefficient.

From now on we assume that the probability of being ranked first $q(x)$ is given by the generalised Tullock success function $q(x_i, r) = \frac{1}{1+x_i^{-r}}$, $i = \{1, 2\}$ and $j = 3 - i$.[16] Moreover, individual information emission cost is assumed to be quadratic $c_i(|\varepsilon_i|) = \varepsilon^2/2$. Notice that for these particular quadratic cost functions Lemma 1 implies the equilibrium identity

$$r^* = \varepsilon_1^* + \varepsilon_2^* = P'^1(r) + P'^2(r). \tag{20}$$

In the first version of our precision contest, we set $P^2 = 0$ and study the players' behaviour under a single, constant prize P^1. Player i's problem is then to

$$\max_{\varepsilon_i} u_i(\theta, \varepsilon) = q(x_i, r)P^1 - c(|\varepsilon_i|) = \frac{1}{1 + x_i^{-r}} P^1 - \frac{\varepsilon_i^2}{2} \tag{21}$$

with focs

$$\varepsilon_1 = P^1 \frac{x^r \log(x)}{(1 + x^r)^2} = -\varepsilon_2. \tag{22}$$

Inserting $r = 0$ from lemma 1 results in the bidding functions

$$\varepsilon_1^* = P^1 \frac{\log(x)}{4} = -\varepsilon_2^*. \tag{23}$$

Proposition 2. *Provided that $x = \theta_1/\theta_2 \leq e^{\sqrt{\frac{2}{P^1}}\,3^{3/4}}$, the contest* (21) *with exogenous prize P^1 has the asymmetric pure strategy equilibrium* (23).[17]

This result confirms the intuition that, with identical (marginal) cost, whatever the higher quality firm invests in increased ranking precision is undone by the low quality firm. Under a status quo of $r = 0$, players thus engage in a pure chance draw and obtain symmetric utility.[18] (The results are completely unaffected if some constant ex-ante precision ε_0 is included in the sum determining the ranking precision.) Equilibrium existence depends on the contest not to be too asymmetric; if x increases beyond the above sufficient condition then pure strategy equilibrium existence becomes problematic and eventually fails. Matching the disinformation a very low-quality player produces in his attempts to 'level the playing field' becomes too costly to allow for positive payoffs of the high-quality player. Therefore, a high quality player informs less than suggested by the prescribed equilibrium. Extremely asymmetric precision contests may therefore not exhibit a pure strategy

[16] We define $q(0/0, r) = 1/2$ and $q(\theta_i/0, r) = 1$ for $\theta_i > 0$ for completeness.

[17] Throughout the paper, e stands for (Euler's) exponential constant, approximately equal to 2.71828.

[18] For a variant where information policies do not fully cancel each other out consider, for instance,

$$\max_{\varepsilon_i} u_i(\theta, \varepsilon) = \frac{1}{1 + x_i^{-r}} P^1 - \frac{\varepsilon_i^2}{2x_i}$$

which leads to the necessary condition $\varepsilon_i^* = -x_i^2 \varepsilon_j^*$. Already this simple variant seems analytically intractable.

equilibrium.

In the next version of our precision contest, we study the players' behaviour when the single prize $P^1(r)$ reacts to the amount of information produced in a linear fashion $ur = u(\varepsilon_1 + \varepsilon_2)$, $u > 0$. The players' problem is to

$$\max_{\varepsilon_i} u_i(\theta, \varepsilon) = q(x_i, r)P^1(r) - c(|\varepsilon_i|) = \frac{1}{1 + x_i^{-r}} ur - \frac{\varepsilon_i^2}{2}. \tag{24}$$

Taking the derivatives wrt ε_i gives the players' focs implicitly as

$$\varepsilon_1 = u\frac{x^r\left(1 + x^r + r\log(x)\right)}{\left(1 + x^r\right)^2}, \ \varepsilon_2 = u\frac{1 + x^r - rx^r\log(x)}{\left(1 + x^r\right)^2} \tag{25}$$

which, by applying the insight from Lemma 1 that $r = \varepsilon_1 + \varepsilon_2 = u$ in equilibrium, we obtain equilibrium bids as

$$\varepsilon_1^* = \frac{ux^u(1 + x^u + u\log(x))}{(1 + x^u)^2}, \ \varepsilon_2^* = \frac{u(1 + x^u - ux^u\log(x))}{(1 + x^u)^2} \tag{26}$$

which are independent of r. Below figure 4 graphs these as functions of x for $u = 2$. The following

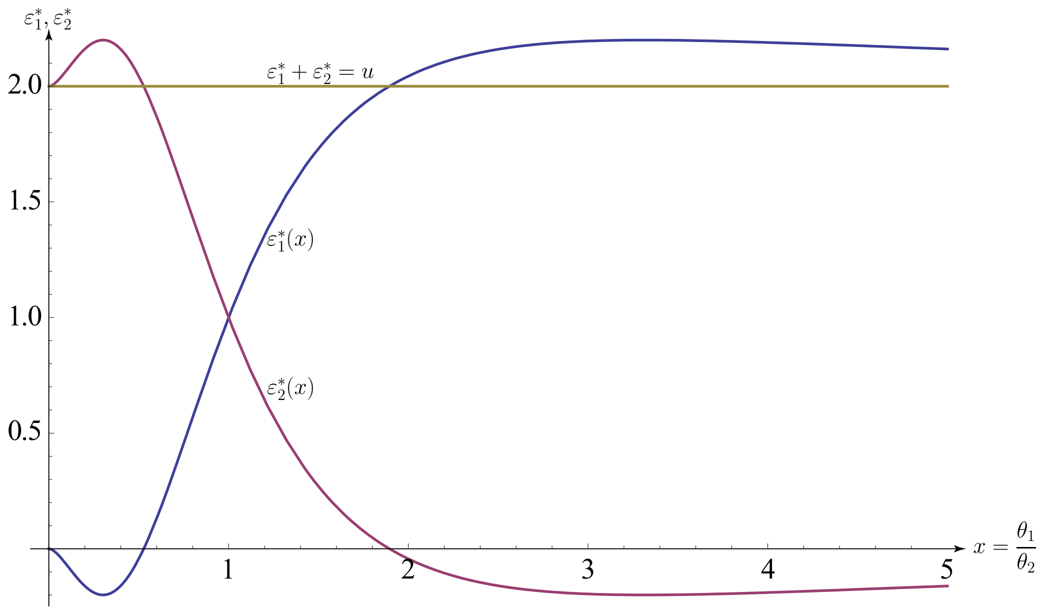


Figure 3: Firms' information dissemination policies as functions of their quality ratio θ_1/θ_2.

proposition shows that this derivation does not depend on the linearity of prize $P^1(r)$.

Proposition 3. *Consider contest* (24)*. For any positive, weakly concave and twice continuously differentiable prize* $P^1(\varepsilon_1 + \varepsilon_2)$ *of the form* $u(\varepsilon_1 + \varepsilon_2)^d$, $0 < d \leq 1$, *we obtain the pair of equilibrium bidding functions*

$$\varepsilon_1^* = u\frac{r^{d-1}x^r(d(x^r + 1) + r\log(x))}{(1 + x^r)^2}, \ \varepsilon_2^* = u\frac{r^{d-1}(d + x^r(d - r\log(x)))}{(1 + x^r)^2} \tag{27}$$

where, in equilibrium, $r = \varepsilon_1^* + \varepsilon_2^* = (ud)^{\frac{1}{2-d}}$ *from lemma 1.*

Notice that concavity in the above proposition is only a sufficient condition. The only requirement is that the derivative of the prize function has a fixed point $P'(r) = r$. There are many examples of convex functions with this property.

It can be easily verified that the limits of the above equilibrium bid functions are given by

$$\varepsilon_1^*(0) = 0,\ \varepsilon_2^*(0) = r^*,\ \lim_{x\to\infty} \varepsilon_1^*(x) = r^*,\ \lim_{x\to\infty} \varepsilon_2^*(x) = 0. \tag{28}$$

Notice that, apart for the limits and for a linear prize, for $x \notin [\text{ProductLog}(\frac{1}{e})^{1/u}, \text{ProductLog}(\frac{1}{e})^{-1/u}] (\approx [0.53, 1.89]$ for $u = 2)$, we find that $\varepsilon_i^*(x) \notin [0, 1]$ although $\varepsilon_1 + \varepsilon_2 \equiv u$ everywhere.[19] For given x, the equilibrium bids turn out to be non-monotonic in u.

Consider now the two-prize extension of this precision contest. As for the first prize $P^1(r) = ur^d$, assume that the second prize is concave in r and given by $P^2(r) = tr^d$, $u > t \geq 0$, $0 < d \leq 1$. A player then faces the problem

$$\max_{\varepsilon_i} u_i(\theta, \varepsilon) = q(x_i, r)P^1(r) + (1 - q(x_i, r))P^2(r) - c(|\varepsilon_i|) = \frac{ur^d}{1 + x_i^{-r}} + \frac{tr^d}{1 + x_i^r} - \frac{\varepsilon_i^2}{2}. \tag{29}$$

Taking derivatives gives the focs which, by substituting $r = (d(t+u))^{\frac{1}{2-d}}$ (from lemma 1) in equilibrium, transform into the equilibrium bids of

$$\begin{aligned} \varepsilon_1^* &= \frac{r^{d-1}\left(d\left(1+x^r\right)\left(t+ux^r\right) - r(t-u)x^r\log(x)\right)}{\left(1+x^r\right)^2}, \\ \varepsilon_2^* &= \frac{r^{d-1}\left(d\left(1+x^r\right)\left(u+tx^r\right) + r(t-u)x^r\log(x)\right)}{\left(1+x^r\right)^2}. \end{aligned} \tag{30}$$

Proposition 4. *Consider contest* (29)*. For any positive, weakly concave and twice continuously differentiable prize differential* $P^1(\varepsilon_1 + \varepsilon_2) - P^2(\varepsilon_1 + \varepsilon_2)$ *of the form* $(u - t)(\varepsilon_1 + \varepsilon_2)^d$, $0 < d \leq 1$, *we obtain the equilibrium bidding functions* (30) *where, in equilibrium,* $r = \varepsilon_1 + \varepsilon_2 = (d(t + u))^{\frac{1}{2-d}}$ *from lemma 1.*

4.1 The full application

In our application, the two prizes derived from the consumer side are non-linear in r and given by

$$P^1(r) = \frac{4\left(2^r - 1\right)\left(1 + 2^{r+1}\right)^2 s^3}{3\left(1 + 2^r\right)\left(2 + 7\times 2^r\right)^2},\ P^2(r) = \frac{\left(2^r - 1\right)\left(2 + 2^r\right)\left(1 + 2^{r+1}\right)s^3}{3\left(1 + 2^r\right)\left(2 + 7\times 2^r\right)^2}. \tag{31}$$

This gives rise to the players' problem

$$\max_{\varepsilon_i} \frac{1}{1 + x_i^{-r}} P^1(r) + \frac{1}{1 + x_i^r} P^2(r) - \frac{\varepsilon_i^2}{2} \tag{32}$$

[19] The ProductLog (also known as Lambert-W or Omega) function gives the principal solution for w in $z = we^w$.

with foc

$$\varepsilon_i = \frac{x^r \log(x)(P^1(r) - P^2(r)) + (1 + x^r)\left(x^r P'^1(r) + P'^2(r)\right)}{(1 + x^r)^2}. \tag{33}$$

We transform the foc into bidding functions for the equilibrium r^* given by Lemma 1 as solution to the fixed point problem[20]

$$r = P'^1(r) + P'^2(r) = \frac{2^{6+r}\left(22 + 81 \times 2^r + 27 \times 4^{r+1} + 59 \times 8^r\right)\log(2)}{(1 + 2^r)^2\,(2 + 7 \times 2^r)^3} \approx 1.7593. \tag{34}$$

For the full application, this polynomial cannot be solved analytically. Inserting the obtained r^* into the foc (33) we get the *linearised* equilibrium information policies

$$\begin{aligned}
\varepsilon_1^* &= \frac{P'^2(r^*) + P'^1(r^*)x^{2r^*} + x^{r^*}\left(P'^1(r^*) + P'^2(r^*) + (P^1(r^*) - P^2(r^*))\log(x)\right)}{\left(1 + x^{r^*}\right)^2}, \\
\varepsilon_2^* &= \frac{P'^1(r^*) + P'^2(r^*)x^{2r^*} + x^{r^*}\left(P'^1(r^*) + P'^2(r^*) + (P^2(r^*) - P^1(r^*))\log(x)\right)}{\left(1 + x^{r^*}\right)^2}.
\end{aligned} \tag{35}$$

Using our example parameters $s = 4$ and $x = 3$, these reproduce the previously obtained values of

$$\varepsilon_1^* = 1.847, \varepsilon_2^* = -0.088. \tag{36}$$

Notice that, since our analysis is independent of the particular functional form of prizes in (31), we can also model different markets such as the downloading of software or advertising without any change to the main analysis above. All that changes are (31) and their derivatives. To give an example of both market types, consider the simple variant of prizes (31)

$$B^1(r) = r^\gamma P^1(r),\ B^2(r) = r^\gamma P^2(r),\ \text{where } \gamma \in \{-\frac{1}{2}, 0, +\frac{1}{2}\} \tag{37}$$

in order to model market demand derived from the same model as above but with an increasing, constant, and decreasing demand aspect of the information provided.[21] The corresponding information output for each parameter value (again for $s = 4$ and $x = 3$) is

$$\begin{aligned}
-{}^1\!/_2: &\quad \varepsilon_1^* = 1.157, \quad \varepsilon_2^* = -0.177, \quad r^* = 0.980, \\
0: &\quad \varepsilon_1^* = 1.847, \quad \varepsilon_2^* = -0.088, \quad r^* = 1.759, \\
{}^1\!/_2: &\quad \varepsilon_1^* = 3.118, \quad \varepsilon_2^* = 0.089, \quad r^* = 3.207.
\end{aligned}$$

Quite intuitively, there is still competition for the larger prize in the shrinking market but the lower quality firm prefers to obfuscate the ranking (e.g., by hiring lawyers to take down ranking sites in the downloading example rather than promoting the high quality product through freely available software). The second line corresponds to our initial example without direct demand impact of information. In the growing market, both firms choose to 'advertise' and expand demand although

[20] Since both $P'^1(r)$ and $P'^2(r)$ are convex and downward sloping for $r > 0$, their sum is convex and downward sloping, too. Since $P'^1(0)+P'^2(0) > 0$, there is a unique fixed point giving a unique equilibrium contest precision.

[21] The possible parameter set for γ is chosen such as not to impinge on equilibrium existence in our example.

the higher quality firm has much more incentives to do so than the lower quality firm.

Finally, we would like to point to another consequence of lemma 1. The interaction we model is a dynamic game where consumers potentially react by updating their prior beliefs on the distribution of qualities F after observing the total market information r^*. Since r^* turns out to be constant in equilibrium for any ratio of qualities x, however, this updating step is trivial and consumers leave their priors unchanged.[22]

5 Combined choice of quality and precision

We now generalise the analysis from section 2 to allow for the combined choice of quality and precision. This is difficult in the full setup of section 2. In order not to cloud the intuition behind competition along both dimensions overly with technical details, we analyse two simplified model variants. The first model is symmetric with a single prize (and has applications in its own right) and quality and precision are chosen simultaneously. The second model analyses the two-prizes version of the under the sequential choice of first quality and then ranking precision.

5.1 Symmetric firms

In the first combined model variant, ex-post symmetric firms choose qualities and ranking precisions simultaneously. For simplicity, we choose a prize which is linear in r. More precisely, we assume that firms $i \in \mathcal{N}$ choose

$$\max_{\theta_i,\varepsilon_i} \frac{1}{1+x_i^{-r}} P^1(r) - \frac{\theta_i^2}{2} - \frac{\varepsilon_i^2}{2},\ r = \varepsilon_i + \varepsilon_j,\ j = i-3 \tag{38}$$

where the prize $P^1(r) = u(r)$ satisfies the assumptions from proposition 3. The first order conditions wrt θ_1, θ_2, ε_1, and ε_2 are

$$\theta_1^2 = \theta_2^2 = \frac{r^2 u x^r}{(1+x^r)^2},\ \varepsilon_1 = u\frac{x^r\,(1+x^r+r\log(x))}{(1+x^r)^2},\ \varepsilon_2 = u - \varepsilon_1. \tag{39}$$

In equilibrium $r = u$ and, for symmetric firms, $x = 1$. This gives equilibrium behaviour as

$$\theta_1^* = \theta_1^* = \frac{u\sqrt{u}}{2} \text{ and } \varepsilon_1^* = \varepsilon_2^* = \frac{u}{2}. \tag{40}$$

Figure 4 verifies this candidate as symmetric pure strategy equilibrium $(\theta^*, \varepsilon^*)$.[23] As usual, pure strategy equilibrium existence in the quality contest may be problematic for high r. It may not be too surprising that (40) does not describe equilibrium behaviour if a firm can deviate in *both* strategic variables simultaneously.

[22] We assume that priors remain unchanged following out of equilibrium firm behaviour.

[23] For $u = 2$, the equilibrium values for the graphed example of figure 4 are $\theta^* = 1.414$ and $\varepsilon^* = 1$.

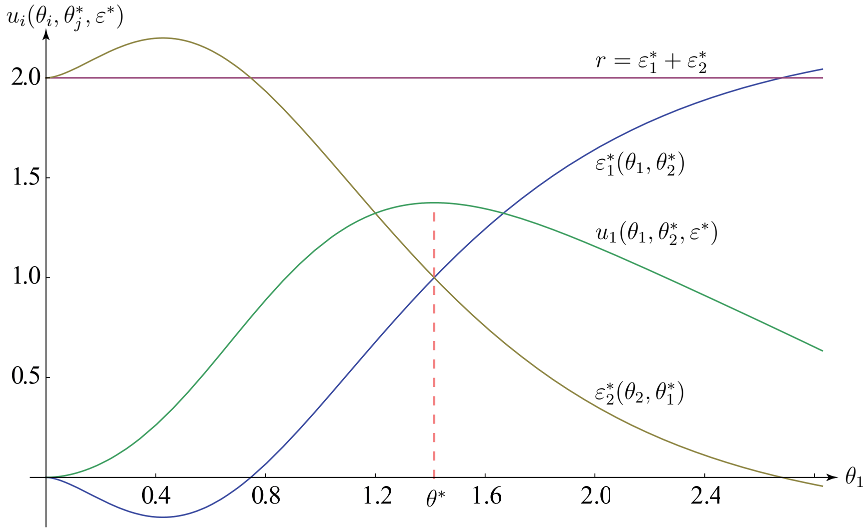


Figure 4: The optimal choice of qualities θ_i in symmetric pure strategy equilibrium.

5.2 Asymmetric firms

In the second model variant, asymmetric firms first choose qualities and then, after observing and learning mutual qualities, they choose the ranking precision. Because the production of quality is now endogenous, this entails some quadratic, idiosyncratic cost of quality. Assume that qualities $\theta_i \in (0, \infty]$ are simultaneously chosen at the first stage and precisions $\varepsilon_i \in \mathbb{R}$ are chosen simultaneously at the second stage for given qualities θ_1 and θ_2. We use backward induction to solve this problem. Precisions $(\varepsilon_1, \varepsilon_2)$ are the result of stage two maximisation

$$\max_{\varepsilon_i} \frac{1}{1+x_i^{-r}} P^1(r) + \frac{1}{1+x_i^{r}} P^2(r) - \frac{\varepsilon_i^2}{2}, \; r = \varepsilon_i + \varepsilon_j, \; j = i - 3 \tag{41}$$

where prizes $P^1(r)$ and $P^2(r)$ are linear in r and satisfy the assumptions from proposition 4. This problem is captured by proposition 4 which gives equilibrium bids (as a function of x) by (30) which we adopt for linear prizes and equilibrium $r^* = u + t$ as

$$\varepsilon_1^* = \frac{t + ux^{2r} + x^r(t + u + r(u - t)\log(x))}{(1 + x^r)^2}, \; \varepsilon_2^* = \frac{u + tx^{2r} + x^r(t + u + r(t - u)\log(x))}{(1 + x^r)^2}.$$

Using these stage two results, we proceed to the first stage where players solve the problem

$$\max_{\theta_i} \frac{1}{1+x_i^{-r^*}} P^1(r^*) + \frac{1}{1+x_i^{r^*}} P^2(r^*) - \frac{\theta_i^2}{2\gamma_i}, \; \gamma_1 = 1, \; \gamma_2 \in (0, \infty). \tag{42}$$

Taking derivatives wrt θ_i we obtain the necessary conditions

$$\theta_1^2 = \frac{rx^r(P1(r) - P2(r))}{(1 + x^r)^2}, \; \theta_2^2 = \frac{rx^r\gamma(P1(r) - P2(r))}{(1 + x^r)^2} \tag{43}$$

implying the equilibrium quality ratio of $x = \theta_1/\theta_2 = \gamma^{-\frac{1}{4}}$. Using this, the above simplify to the equilibrium bidding functions of

$$\theta_1^* = \frac{\sqrt{u-t}(t+u)\gamma^{\frac{t+u}{8}}}{1+\gamma^{\frac{t+u}{4}}}, \ \theta_2^* = \frac{\sqrt{u-t}(t+u)\gamma^{\frac{4+t+u}{8}}}{1+\gamma^{\frac{t+u}{4}}}. \tag{44}$$

Figure 5 verifies this candidate as pure strategy equilibrium $((\theta_1^*, \theta_2^*), (\varepsilon_1^*, \varepsilon_2^*))$ for parameters $u = 4/3$, $t = 2/3$, and $\gamma_2 = 2$.[24]

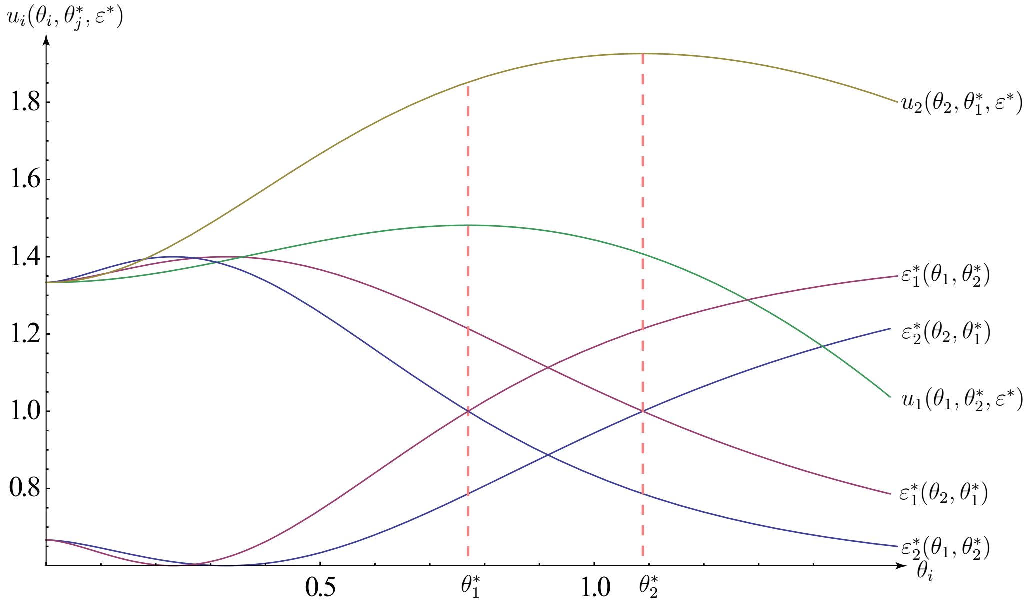


Figure 5: The optimal choice of qualities θ_i in asymmetric pure strategy, subgame perfect equilibrium.

6 Efficiency and welfare analysis

The introduction of labelling into a credence market is not innocuous. If, for instance, two firms are in full price competition in an unlabelled credence market, the introduction of a single label will create two separated monopoly markets. While consumer welfare is maximal in the first market it is minimised in the second. For the purposes of making comparisons between these market types we make the following assumption.

Assumption. *If more than one products are given identical labels, then whatever level of competition governs the original, unlabelled credence market also applies to this 'over-labelled' market (segment).*

Our measure of inefficiency in this type of market corresponds to the mass of consumers who are excluded from consumption although they value the good higher than its marginal production cost of zero. As labelling gives consumers the ability to choose among differentiated products, we have two consumer welfare effects pulling in opposite directions: consumer welfare is improved

[24] The equilibrium values for the graphed example of figure 5 are $\theta^* = (0.804, 1.138)$ and $\varepsilon^* = (0.887, 1.113)$.

through the increased differentiation but decreased through the firms' ability to charge monopoly prices. Producer welfare is unambiguously improved through the introduction of labelling. In order to discuss the social repercussions of introducing a ranking quantitatively, this section formalises a welfare function. The discussion takes the ex-ante point of view, that is, we evaluate consumers' and producers' expectations before any private information becomes available.

6.1 Benchmark case: the unlabelled market

For a pure credence good and in the absence of labelling, consumers expect a pooled quality of $\mathbb{E}[\Theta]$ to which cartelised firms optimally set a pooled price of $\bar{p} = p_1 = p_2$.[25] This expected pooled quality type is given by

$$\mathbb{E}[\Theta] = \int_0^s f(\theta) d(\theta) = \mathbb{E}[\Theta_{(1:1)}]. \tag{45}$$

Then the indifferent, lowest participating consumer 'type' $\underline{\mu}$ is identified through

$$v(\cdot) = \underline{\mu}\,\mathbb{E}[\Theta] - \bar{p} = 0 \Leftrightarrow \underline{\mu} = \frac{\bar{p}}{\mathbb{E}[\Theta]} > 0. \tag{46}$$

Thus, a cartel acting as single, multi-product monopolist chooses $\bar{p}$ to maximise its surplus

$$\bar{\mathsf{PS}}_m = \bar{p} \int_{\underline{\mu}}^s g(\mu) d\mu \;\; \Leftrightarrow \bar{p} = \mathbb{E}[\Theta] \frac{1 - G(\underline{\mu})}{g(\underline{\mu})} > 0 \tag{47}$$

while consumers enjoy

$$\bar{\mathsf{CS}}_m = \int_{\underline{\mu}}^s \left(\mu\,\mathbb{E}[\Theta] - \bar{p}\right) g(\mu) d\mu. \tag{48}$$

Total welfare under fully colluding firms is then given by $\bar{W}_m = \bar{\mathsf{PS}}_m + \bar{\mathsf{CS}}_m$. Competitive firms, however, have incentives to decrease their price under $\bar{p}$ in order to capture the full demand. This Bertrand undercutting strategy leads to $\bar{p} = \underline{\mu} = 0$ in (47), implying zero profits and resulting in consumers obtaining the full surplus

$$\bar{\mathsf{CS}}_b = \mathbb{E}[\Theta] \int_{\underline{\mu}}^s \mu g(\mu) d\mu = \bar{\mathsf{W}}_b \text{ and } \bar{\mathsf{PS}}_b = 0. \tag{49}$$

The actually incurred welfare in the unlabelled credence market lies in between these limiting cases and depends on the industry's degree of competitiveness.[26]

[25] In the present analysis we assume that this is true. It is not hard, however, to underpin this with a simple symmetric model which produces this equilibrium behaviour.

[26] Nearly all industries mentioned in the introduction exhibit some form of price setting behaviour. For books and DVDs, there are pre-printed recommended retail prices, (UK) university fees are capped at some typically charged amount, doctors have standard fees for their services etc.

6.2 The fully labelled market

We define a fully labelled market as one awarding a distinct label to each product. In a fully labelled 2-product market, ex-ante expected producer surplus is symmetric and given by

$$\mathsf{PS}_{\#1} + \mathsf{PS}_{\#2} = P^1(r) + P^2(r) - c(\tilde{x}, r^*),\ c(\tilde{x}, r^*) = \frac{\varepsilon_1^*(\tilde{x}_1)^2}{2} + \frac{\varepsilon_2^*(\tilde{x}_2)^2}{2} \tag{50}$$

for expected ratios $\tilde{x}_1 = \mathbb{E}[\Theta_{(1:2)}]/\,\mathbb{E}[\Theta_{(2:2)}]$ and $\tilde{x}_2 = \mathbb{E}[\Theta_{(2:2)}]/\,\mathbb{E}[\Theta_{(1:2)}]$, prizes $P(r)$ come from (8), $r^* = P'^1(r^*) + P'^2(r^*)$ given by (20), and ε_i^* from (35). Recall that consumers expect ranked qualities Λ (4) and winning probabilities $\tilde{q}$ (5). Similarly, expected consumer surplus is the sum of

$$\mathsf{CS}_{\#1} = \int_{\hat{\mu}_2^1}^{\hat{\mu}_1^0} \left(\mu\Lambda^1 - p_1\right) g(\mu) d\mu,\ \mathsf{CS}_{\#2} = \int_{\hat{\mu}_3^2}^{\hat{\mu}_2^1} \left(\mu\Lambda^2 - p_2\right) g(\mu) d\mu \tag{51}$$

with equilibrium cutoffs $\hat{\mu}$ and (monopoly) prices p defined in (7) and (8). Notice that—although the welfare components (50) and (51) may be involved—the combined welfare for each labelled market segment is given by the simple measure

$$\mathsf{W}_{\#} = \mathsf{CS}_{\#} + \mathsf{PS}_{\#} = \int_{\hat{\mu}_2^1}^{\hat{\mu}_1^0} \mu\, \mathbb{E}[\Theta_{(1:2)}] g(\mu) d\mu + \int_{\hat{\mu}_3^2}^{\hat{\mu}_2^1} \mu\, \mathbb{E}[\Theta_{(2:2)}] g(\mu) d\mu - c(r^*). \tag{52}$$

Under the uniform distribution and for $\tilde{q}^2 = \left(\frac{1}{1+x^r}, \frac{1}{1+x^{-r}}\right)$ defined in (5), we obtain

$$0 < \hat{\mu}_3^2 = \frac{p_2^*}{\Lambda^2} = \frac{p_2}{\tilde{q}_1^2\, \mathbb{E}[\Theta_{(1:2)}] + \tilde{q}_2^2\, \mathbb{E}[\Theta_{(2:2)}]} < \frac{\bar{p}}{\mathbb{E}[\Theta]} = \underline{\mu}, \tag{53}$$

because, in equilibrium, $\underline{\mu} = \frac{s}{2}$ and $\hat{\mu}_3^2 = \frac{\hat{\mu}_2^1}{2}$ with $\hat{\mu}_2^1 < s$.[27] Thus, the lowest cutoff in the labelled market is unambiguously smaller than the cutoff in the unlabelled market $\underline{\mu}$ from (46) and the mass of consumers excluded from consumption in the labelled market is strictly smaller than in the unlabelled market. This property generalises to the following result on market efficiency.

Proposition 5. *For any number of firms $n = |\mathcal{N}|$, if the distribution of consumer tastes $G(\mu)$ satisfies the condition*

$$\frac{G(\hat{\mu}_n^{n-1}) - G(\hat{\mu}_{n+1}^n)}{g(\hat{\mu}_{n+1}^n)} < \frac{1 - G(\underline{\mu})}{g(\underline{\mu})} \tag{54}$$

on the lowest equilibrium cutoff $\hat{\mu}_{n+1}^n$, then the fully labelled and cartelised market is strictly more efficient than its unlabelled equivalent under equilibrium cutoff $\underline{\mu}$.

We now turn to the comparison of social welfare in the two market types. In the unlabelled market, the welfare in the monopoly and fully competitive Bertrand equilibrium cases are given by

$$\bar{\mathsf{W}}_m = \int_{\underline{\mu}_m}^{\hat{\mu}_1^0} \mathbb{E}[\Theta]\mu g(\mu) d\mu \quad \text{and} \quad \bar{\mathsf{W}}_b = \int_{\underline{\mu}_b}^{\hat{\mu}_1^0} \mathbb{E}[\Theta]\mu g(\mu) d\mu \tag{55}$$

[27] This follows from the fact that stochastic orders are equally spaced for the uniform distribution.

where (as in the previous subsection) we use subscripts m, b to discriminate between the monopoly pricing and Bertrand undercutting policies, respectively. Note that $\underline{\mu}_m > \underline{\mu}_b = 0$ which lets us transform the above into

$$\bar{\mathsf{W}}_m = \mathbb{E}[\Theta](\mathbb{E}[\mu] - \int_{\underline{\mu}_b=0}^{\underline{\mu}_m} \mu g(\mu) d\mu) \text{ and } \bar{\mathsf{W}}_b = \mathbb{E}[\Theta]\,\mathbb{E}[\mu]. \tag{56}$$

Thus, little surprisingly, the unlabelled Bertrand welfare is strictly above the welfare under cartelised prices $\bar{\mathsf{W}}_b > \bar{\mathsf{W}}_m$. We rewrite the unlabelled Bertrand welfare $\bar{\mathsf{W}}_b$ as

$$\bar{\mathsf{W}}_b = \int_{\underline{\mu}_b}^{\hat{\mu}_3^2} \mathbb{E}[\Theta]\mu g(\mu) d\mu + \int_{\hat{\mu}_3^2}^{\hat{\mu}_2^1} \mathbb{E}[\Theta]\mu g(\mu) d\mu + \int_{\hat{\mu}_2^1}^{\hat{\mu}_1^0} \mathbb{E}[\Theta]\mu g(\mu) d\mu \tag{57}$$

and subtract the preceding expression from the simplified form of expected welfare $\mathsf{W}_{\#}$ in the labelled market (52) to obtain $\mathsf{W}_{\#} - \bar{\mathsf{W}}_b$ as

$$\int_{\hat{\mu}_1^2}^{\hat{\mu}_1^0} \mu(\mathbb{E}[\Theta_{(1:2)}] - \mathbb{E}[\Theta])g(\mu) d\mu + \int_{\hat{\mu}_3^2}^{\hat{\mu}_2^1} \mu(\mathbb{E}[\Theta_{(2:2)}] - \mathbb{E}[\Theta])g(\mu) d\mu - \int_{\underline{\mu}_b}^{\hat{\mu}_3^2} \mathbb{E}[\Theta]\mu g(\mu) d\mu - c(r^*).$$

Notice that total welfare in the labelled market is weakly higher than in its unlabelled equivalent as the planer can always give the same label to the two firms reproducing the competitive situation of the unlabelled market and thus generating the same welfare. Moreover, for more than two firms, total welfare in the labelled market is strictly higher than in its unlabelled counterpart as the planer can award two labels to the two highest-ranked firms which would then undercut each other until reaching marginal cost $p = 0$. Thereby they drive out all lower quality firms and thus improve consumer welfare. After some manipulation of the above expression (see proof for details), we arrive at the following proposition.

Proposition 6. *If only a single label is awarded per firm, then total welfare in the fully labelled market is strictly higher than in the unlabelled market if the following condition is respected*

$$\frac{\mathbb{E}[\Theta_{(1:2)}] - \mathbb{E}[\Theta]}{\mathbb{E}[\Theta]} > \frac{\mathbb{E}[\mu|\hat{\mu}_3^2 > \mu > \underline{\mu}_b] + \frac{c(r)}{\mathbb{E}[\Theta]}}{(\mathbb{E}[\mu|\hat{\mu}_1^0 > \mu > \hat{\mu}_2^1] - \mathbb{E}[\mu|\hat{\mu}_2^1 > \mu > \hat{\mu}_3^2])}. \tag{58}$$

To illustrate this result we continue our example from section 3. There, the (monopoly) prices which firms charge give a consumer surplus of

$$\mathsf{CS}_{\#m} = \frac{(14 + 13 \times 2^r)(1 + 2^{r+1})^2 s^3}{6(2^r + 1)(2 + 7 \times 2^r)^2} \tag{59}$$

for which we can derive the following numerical relationships

$$\underbrace{\mathsf{CS}_{Pm}}_{=s^3\frac{1}{16}} < \underbrace{\lim_{r\to\infty} \mathsf{CS}_{\#m}}_{=s^3\frac{26}{147}} < \mathsf{CS}_{\#m} < \underbrace{\mathsf{CS}_{Pb}}_{=s^3\frac{1}{4}}. \tag{60}$$

Figure 6 shows the relation between these for different levels of industry competition. As above, subscripts $\{b, m\}$ denote the cases of Bertrand and monopolistic competition, respectively. Subscript $\#$ denotes a surplus under the ranking while subscript P denotes fully pooled quality expectations. Numerical results for $n > 2$ products confirm these relationships.

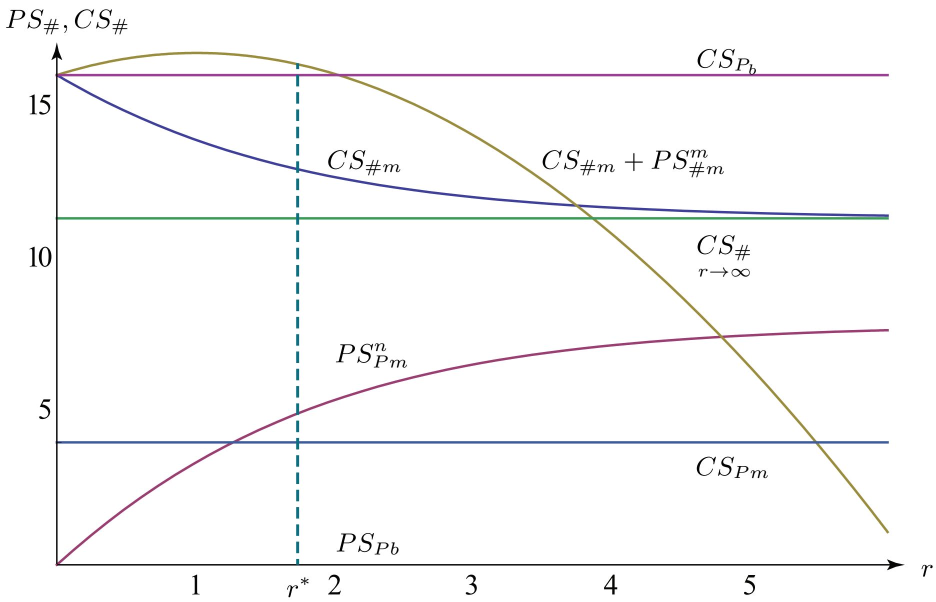


Figure 6: The introduction of labelling is welfare improving unless the industry is fully Bertrand-competitive.

7 Concluding remarks

We present the first analysis of contest models dealing with endogenous ranking (or labelling) precisions. As this precision is endogenously generated, we effectively allow players to choose the ranking technology for their market. We obtain a tractable analytical description of the firms' information dissemination functions together with first and tentative efficiency and welfare results. In a first application of this theory we unify the consumer and producer side in a simple oligopoly model parameterised in all strategic choices by the endogenous ranking precision. We hope that our simple model is capable of illuminating the workings of at least some of the many credence markets we deem applicable to this setting.

Appendix

Proof of proposition 1. We wish to establish that, for any given vector of prices $(p_1, \ldots, p_n)$ for competing products, the revenue function of firm k, $1 \leq k \leq n$, is a continuous single peaked (quasi-concave) function of its price p_k. The existence of an equilibrium then follows immediately from standard arguments. First see that the first derivative equals to

$$\frac{\partial P^k}{\partial p_k} = \int_{\hat{\mu}_{k+1}^k}^{\hat{\mu}_k^{k-1}} g(\mu)d\mu - \frac{p_k\left(g(\hat{\mu}_k^{k-1}) + g(\hat{\mu}_{k+1}^k)\right)}{\Lambda^k - \Lambda^{k+1}} = 0. \tag{61}$$

Underlying that

$$p_k^* = \frac{(\Lambda^k - \Lambda^{k+1})\left(\int_{\hat{\mu}_{k+1}^k}^{\hat{\mu}_k^{k-1}} g(\mu)d\mu\right)}{\left(g(\hat{\mu}_k^{k-1}) + g(\hat{\mu}_{k+1}^k)\right)}. \tag{62}$$

The second derivatives leads to the following equations

$$\frac{\partial^2 P^k}{\partial^2 p_k} = -\frac{2(\Lambda^k - \Lambda^{k+1})\left(g(\hat{\mu}_k^{k-1}) + g(\hat{\mu}_{k+1}^k)\right) + p_k\left(-g'(\hat{\mu}_k^{k-1}) + g'(\hat{\mu}_{k+1}^k)\right)}{(\Lambda^k - \Lambda^{k+1})^2}. \tag{63}$$

Substituting by p_k and assuming that the distribution satisfies the condition that $\forall \mu \in [0, b]$ with $b \in [0, s[$ we have

$$g(\mu)^2 - g'(\mu)\int_{\mu}^{b} g(x)dx > 0, \tag{64}$$

which implies (9), we can prove that P^k is a concave function at that point which is therefore a local maximum

$$\begin{gathered}
2(\Lambda^k - \Lambda^{k+1})\left(g(\hat{\mu}_k^{k-1}) + g(\hat{\mu}_{k+1}^k)\right) + p_k\left(-g'(\hat{\mu}_k^{k-1}) + g'(\hat{\mu}_{k+1}^k)\right) > 0 \\
2(\Lambda^k - \Lambda^{k+1})\left(g(\hat{\mu}_k^{k-1}) + g(\hat{\mu}_{k+1}^k)\right) > p_k\left(g'(\hat{\mu}_k^{k-1}) - g'(\hat{\mu}_{k+1}^k)\right) \\
2(\Lambda^k - \Lambda^{k+1})\left(g(\hat{\mu}_k^{k-1}) + g(\hat{\mu}_{k+1}^k)\right) > \frac{(\Lambda^k - \Lambda^{k+1})\left(G(\hat{\mu}_k^{k-1}) - G(\hat{\mu}_{k+1}^k)\right)}{\left(g(\hat{\mu}_k^{k-1}) + g(\hat{\mu}_{k+1}^k)\right)}\left(g'(\hat{\mu}_k^{k-1}) - g'(\hat{\mu}_{k+1}^k)\right) \\
2\left(2g(\hat{\mu}_{k+1}^k)g(\hat{\mu}_k^{k-1}) + g(\hat{\mu}_{k+1}^k)^2\right)^2 + g'(\hat{\mu}_{k+1}^k)\left(G(\hat{\mu}_k^{k-1}) - G(\hat{\mu}_{k+1}^k)\right) \\
+ \underbrace{g(\hat{\mu}_k^{k-1})^2 - \left(G(\hat{\mu}_k^{k-1}) - G(\hat{\mu}_{k+1}^k)\right)g'(\hat{\mu}_k^{k-1})}_{>0} > 0.
\end{gathered}$$

Thus any distribution G which satisfies (9) leads to the claimed equilibrium behaviour.[28] □

Proof of lemma 1. Given the objective (1), the two focs wrt ε_i, $i = 1, 2$ are

$$c'(|\varepsilon_1|) - P'^2(r) = y, \ -c'(|\varepsilon_2|) + P'^1(r) = y \tag{65}$$

where

$$y = q'(r)(P^1(r) - P^2(r)) + q(r)(P'^1(r) - P'^2(r)) \tag{66}$$

The first pair of equations readily transforms into our claim (17). □

Proof of proposition 2. Consider the equilibrium candidate (23). Notice that the positive branch

[28] Note that a sufficient condition for the function P^k to be concave is that $g(\mu)$ is weakly concave as in Bonnisseau and Lahmandi-Ayed (2007). Indeed if $g(\mu)$ is weakly concave, $-g'(\hat{\mu}_k^{k-1}) + g'(\hat{\mu}_{k+1}^k) \geq 0$ as $\hat{\mu}_k^{k-1} > \hat{\mu}_{k+1}^k$ and $g'(\cdot)$ weakly decreasing.

of $q(x,r) = \frac{1}{1+x_i^{-r}}$ is not concave for sufficiently large x and thus the optimisation problem is not well-behaved. We pursue a sufficient condition for $q(x,r)$ to be weakly concave in dependence of the measure of asymmetry x. Given equilibrium behaviour of the opponent, i.e., $\varepsilon_2^* = -P^1 \frac{\log(x)}{4}$, firm 1's payoff

$$u_1(x, \varepsilon_1, \varepsilon_2^*) = \frac{1}{1 + x^{\varepsilon_1 + \varepsilon_2^*}} P^1 - \frac{\varepsilon_1^2}{2} \tag{67}$$

has a curvature (i.e., second derivative wrt ε_1) of

$$\frac{x^{\varepsilon_1+\varepsilon_2^*}\left(1 - x^{\varepsilon_1+\varepsilon_2^*}\right)}{\left(1 + x^{\varepsilon_1+\varepsilon_2^*}\right)^3} P^1 \log(x)^2 - 1 \tag{68}$$

which we want to ensure to be negative. Since $1 - x^r < 0$ for any $r > 0$, we need to be concerned only with the case of $\varepsilon_1 < -\varepsilon_2^*$, i.e., $r < 0$ ($r = r^* = 0$ being the equilibrium candidate). We therefore rearrange and consider the problem of finding x ensuring that

$$\left(1 + x^{\varepsilon_1+\varepsilon_2^*}\right)^3 + P^1 \log(x)^2 x^{\varepsilon_1+\varepsilon_2^*}\left(x^{\varepsilon_1+\varepsilon_2^*} - 1\right) > 0. \tag{69}$$

We take the derivative of the lhs wrt to ε_1 to obtain the critical point[29]

$$3\left(1 + x^{\varepsilon_1+\varepsilon_2^*}\right)^2 + P^1 \log(x)^2 \left(2x^{\varepsilon_1+\varepsilon_2^*} - 1\right) = 0 \tag{70}$$

which is solved by

$$\tilde{\varepsilon}_1 = \frac{\log\left(\frac{1}{3}\left(\log(x)\left(\sqrt{P^1\left(9 + P^1 \log(x)^2\right)} \pm P^1 \log(x)\right) - 3\right)\right)}{\log(x)} - \varepsilon_2^*. \tag{71}$$

Substituting $\varepsilon_2^* = -P^1 \frac{\log(x)}{4}$ and plugging this critical $\tilde{\varepsilon}_1$ back into (69) gives the condition

$$\tilde{x} < e^{\sqrt{\frac{2}{P^1}}\, 3^{3/4}} \tag{72}$$

for strict concavity of (67). The argument for player 2 is symmetric and results in the same condition. □

Proof of proposition 3. Consider the problem of player 1 (24) given equilibrium behaviour of player 2. Player 1's payoff expectation is obviously negative for $r = \varepsilon_1 + \varepsilon_2^* < 0$. For non-negative r, the strategy of the proof is to show that the first derivative of the objective $\frac{ur}{1+x^{-r}} - \frac{\varepsilon_1^2}{2}$

$$\frac{x^{\varepsilon_1+\varepsilon_2^*}(u - \varepsilon_1) - \varepsilon_1}{1 + x^{\varepsilon_1+\varepsilon_2^*}} + \frac{u x^{\varepsilon_1+\varepsilon_2^*}(\varepsilon_1 + \varepsilon_2^*)\log(x)}{\left(1 + x^{\varepsilon_1+\varepsilon_2^*}\right)^2} \tag{73}$$

is positive left of the critical point given by (26) and negative to the right. We will use the fact that the objective consists of a strictly quasi-concave gain function $(ur)/(1+x^{-r})$ with at most one turning point (where the second derivative $r\log(x) = 2(x^r+1)/(x^r-1)$ changes sign) on $r > 0$

[29] In the relevant range $0 \le \varepsilon_1 < -\varepsilon_2^*$, the objective has at most one critical point which is necessarily a maximum.

from which we subtract a strictly convex cost $\varepsilon_1^2/2$ without turning point on the same interval. Hence the objective has at most one turning point for $\varepsilon_1 \geq -\varepsilon_2^*$.

a) $\varepsilon_2^* < 0 \;\;\Leftrightarrow\;\; u > \frac{1+\text{ProductLog}(1/e)}{\log(x)}$: At the point where $\varepsilon_1 = -\varepsilon_2^*$, i.e. $r = 0$, the above first derivative (73) reduces to $u_1'(\varepsilon, x)\big|_{\varepsilon_1=-\varepsilon_2^*} = \frac{u}{2} + \varepsilon_2^*$. For $u > \frac{1+\text{ProductLog}(1/e)}{\log(x)}$ this implies that

$$u_1'(\varepsilon, x)\bigg|_{\varepsilon_1=-\varepsilon_2^*} > \frac{1+\text{ProductLog}(1/e)}{\log(x^2)} \tag{74}$$

for equilibrium ε_2^* from (26); this is positive for $x > 1$. The second derivative at this point evaluates to $\frac{1}{2}u\log(x) - 1$ which may be positive or negative depending on x. a.i) If the first derivative is increasing at $\varepsilon_1 = -\varepsilon_2^*$, since the objective has at most one turning point to the right of this point, the first derivative reaches a maximum at this turning point and then forever falls monotonically intersecting the horizontal at ε_1^*. a.ii) If the first derivative is decreasing at $\varepsilon_1 = -\varepsilon_2^*$, then the objective has no turning point to the right of this point and the first derivative falls monotonically forever changing sign at ε_1^*.

b) $\varepsilon_2^* \geq 0 \;\;\Leftrightarrow\;\; u \leq \frac{1+\text{ProductLog}(1/e)}{\log(x)}$: Since now the first derivative $u_1'(\varepsilon, x) = \frac{u}{2} + \varepsilon_2^*$ can be directly seen to be positive the same argument as above goes through. As the reasoning for player 2 is entirely symmetric, this confirms (26) as pure strategy equilibrium.

Any concave prize transformation of the form

$$\frac{ur^d}{1+x^{-r}},\;\; 0 < d \leq 1, \tag{75}$$

leaves the gains function strictly quasiconcave. Again, the first derivative of the objective (24) is always positive for $-\varepsilon_2^* \leq \varepsilon_1 < \varepsilon_1^*$ defined in (27), and negative for $\varepsilon_1 < \varepsilon_1^*$. As above, the function has at most one turning point. Therefore, exactly the same analysis as above goes through confirming (27) as equilibrium of the contest (24). □

Proof of proposition 4. It is well-known from the contests literature that equilibrium efforts in standard tournaments only depend on the difference between prizes. In a similar manner, we can rewrite the objective of contest (29) as a single-prize contest

$$\frac{(u-t)r^d}{1+x^r} + tr^d - \frac{\varepsilon_1^2}{2}. \tag{76}$$

Since $u - t > 0$ by assumption and an unconditional uniform transfer of tr^d cannot hurt existence, this single-prize contest meets all assumptions of proposition 3. Therefore the existence of equilibrium (30) is ensured. □

Proof of proposition 5. We define total welfare as $\mathsf{W}_\# = \sum_{k=1}^{n} \mathsf{W}_\#^k$ over

$$\mathsf{W}_\#^k = \mathsf{CS}_\#^k + \mathsf{PS}_\#^k = \int_{\hat{\mu}_{k+1}^k}^{\hat{\mu}_k^{k-1}} \mu\, \mathbb{E}[\Theta_{(k:n)}] g(\mu) d\mu - \frac{\varepsilon^*(\tilde{x}_k)^2}{2} \tag{77}$$

where the ratio vector $\tilde{x}_k = \mathbb{E}[\Theta_{(k:n)}]/\,\mathbb{E}[\Theta_{(-k:n)}]$, with $-k$ specifying $(1,\ldots,k-1,k+1,\ldots,n)$, denotes the expectation of the n-player generalisation of $x=\theta_1/\theta_2$ consisting of the $(n-1)$ ratios of $\mathbb{E}[\Theta_{(k:n)}]$ over the other order statistics. Similar to (53) we generalise $\hat{\mu} = (\hat{\mu}_1^0 = 1,\ldots,\hat{\mu}_{n+1}^n)$ with

$$\hat{\mu}_{k+1}^k = \frac{p_k - p_{k+1}}{\Lambda^k - \Lambda^{k+1}},\ \hat{\mu}_{n+1}^n = \frac{p_n}{\Lambda^n} = \frac{p_n^*}{\sum_{i=1}^{n} \tilde{q}_i^n \,\mathbb{E}[\Theta_{(i:n)}]} > 0 \tag{78}$$

where we define the n-players extension of $\tilde{q}_i^k$ (5) by the vector of ratio-dependent probabilities

$$\tilde{q}^1 = (\tilde{q}_1^1(\tilde{x}_1^1,\tilde{x}_2^1,\ldots,\tilde{x}_n^1,r),\tilde{q}_2^1(\tilde{x}_1^2,\tilde{x}_2^2,\ldots,\tilde{x}_n^2,r),\ldots,\tilde{q}_n^1(\tilde{x}_1^n,\tilde{x}_2^n,\ldots,\tilde{x}_n^n,r)) \tag{79}$$

with $\tilde{x}_k^i = \dfrac{\mathbb{E}[\Theta_{(i:n)}]}{\mathbb{E}[\Theta_{(k:n)}]}$. We assume the usual properties of these probabilities; for details see, for instance, Giebe and Schweinzer (2011, section 5.1). We know from (20) that $r^* > 0$ in equilibrium. Therefore, the lowest quality expectation is strictly below the unlabelled market expectation

$$\Lambda^n = \sum_{i=1}^{n} \tilde{q}_i^n \,\mathbb{E}[\Theta_{(i:n)}] < \mathbb{E}[\Theta]. \tag{80}$$

We know moreover that the choice of the optimal (monopoly) price p_n^* for the lowest-labelled market segment

$$\arg\max_{p_n} p_n \int_{\hat{\mu}_{n+1}^n}^{\hat{\mu}_n^{n-1}} g(\mu)d\mu = p_n\left(G(\hat{\mu}_n^{n-1}) - G(\hat{\mu}_{n+1}^n)\right) \tag{81}$$

leads to an equilibrium price

$$p_n^* = \Lambda^n \frac{G(\hat{\mu}_n^{n-1}) - G(\hat{\mu}_{n+1}^n)}{g(\hat{\mu}_{n+1}^n)} \tag{82}$$

and therefore the cutoff

$$\hat{\mu}_{n+1}^n = \frac{G(\hat{\mu}_n^{n-1}) - G(\hat{\mu}_{n+1}^n)}{g(\hat{\mu}_{n+1}^n)}. \tag{83}$$

Comparing this to the unlabelled cutoff (46), we obtain (54) under which the labelled firms serve more consumers than the unlabelled firms. □

Proof of proposition 6. Applying the Chasles relation, welfare in the unlabelled market and undercutting strategy $\bar{\mathsf{W}}_b$ can be rewritten as

$$\bar{\mathsf{W}}_b = \int_{\underline{\mu}_b}^{\hat{\mu}_3^2} \mathbb{E}[\Theta]\mu g(\mu)d\mu + \int_{\hat{\mu}_3^2}^{\hat{\mu}_2^1} \mathbb{E}[\Theta]\mu g(\mu)d\mu + \int_{\hat{\mu}_2^1}^{\hat{\mu}_1^0} \mathbb{E}[\Theta]\mu g(\mu)d\mu. \tag{84}$$

We subtract the preceding expression from the simplified form of expected welfare $\mathsf{W}_\#$ in the labelled market (52) and denote $c(r) = \frac{(\varepsilon_1^*(\tilde{x}_1))^2}{2} + \frac{(\varepsilon_2^*(\tilde{x}_2))^2}{2}$ to obtain

$$\mathsf{W}_\# - \bar{\mathsf{W}}_b = \int_{\hat{\mu}_2^1}^{\hat{\mu}_1^0} \mu(\mathbb{E}[\Theta_{(1:2)}] - \mathbb{E}[\Theta])g(\mu)d\mu + \int_{\hat{\mu}_3^2}^{\hat{\mu}_2^1} \mu(\mathbb{E}[\Theta_{(2:2)}] - \mathbb{E}[\Theta])g(\mu)d\mu - \int_{\underline{\mu}_b}^{\hat{\mu}_3^2} \mathbb{E}[\Theta]\mu g(\mu)d\mu - c(r).$$

From the definition of order statistics we obtain that

$$(\mathbb{E}[\Theta_{(1:2)}] - \mathbb{E}[\Theta]) = -(\mathbb{E}[\Theta_{(2:2)}] - \mathbb{E}[\Theta]) \text{ and } (\mathbb{E}[\Theta_{(2:2)}] < \mathbb{E}[\Theta]). \tag{85}$$

Indeed, the probability density function of the k^{th} highest order statistic among n draws can be written as

$$f_{(k:n)}(x) = n \begin{pmatrix} n-1 \\ k-1 \end{pmatrix} F(x)^{n-k}(1-F(x))^{k-1} f(x). \tag{86}$$

Applying this pdf function to our situation, where $\mathbb{E}[\Theta_{(1:2)}]$ is the expected value of the draw ranked first of the two draws and $\mathbb{E}[\Theta_{(2:2)}]$ is the expected value of the draw ranked second of the two draws. Finally $\mathbb{E}[\Theta]$ can be understood as the draw ranked first of only one draw, i.e. $\mathbb{E}[\Theta_{(1:1)}] = \mathbb{E}[\Theta]$. This gives us the following equation

$$\begin{gathered}
\mathbb{E}[\Theta_{(1:2)}] - \mathbb{E}[\Theta] = \int_0^s 2F(x)f(x)xdx - \int_0^s f(x)xdx \\
-(\mathbb{E}[\Theta_{(2:2)}] - \mathbb{E}[\Theta]) = -\int_0^s 2(1-F(x))f(x)xdx + \int_0^s f(x)xdx \\
\mathbb{E}[\Theta_{(1:2)}] - \mathbb{E}[\Theta] + \mathbb{E}[\Theta_{(2:2)}] - \mathbb{E}[\Theta] = 0 \\
\int_0^s 2F(x)f(x)xdx - \int_0^s f(x)xdx + \int_0^s 2(1-F(x))f(x)xdx - \int_0^s f(x)xdx \\
\int_0^s (2(F(x)) + 2(1-F(x)))f(x)xdx - 2\int_0^s f(x)xdx \\
\int_0^s 2f(x)xdx - 2\int_0^s f(x)xdx = 0
\end{gathered} \tag{87}$$

which proves (85). Then

$$\begin{aligned}
\mathsf{W}_{\#} - \bar{\mathsf{W}}_b &= (\mathbb{E}[\Theta_{(1:2)}] - \mathbb{E}[\Theta]) \left(\int_{\hat{\mu}_1^2}^{\hat{\mu}_1^0} \mu g(\mu) d\mu - \int_{\hat{\mu}_3^2}^{\hat{\mu}_2^1} \mu g(\mu) d\mu \right) - \int_{\underline{\mu}_b}^{\hat{\mu}_3^2} \mathbb{E}[\Theta] \mu g(\mu) d\mu - c(r) \\
&= (\mathbb{E}[\Theta_{(1:2)}] - \mathbb{E}[\Theta]) \left(\mathbb{E}[\mu | \hat{\mu}_1^0 > \mu > \hat{\mu}_1^2] - \mathbb{E}[\mu | \hat{\mu}_2^1 > \mu > \hat{\mu}_3^2]\right) - \mathbb{E}[\Theta]\, \mathbb{E}[\mu | \hat{\mu}_3^2 > \mu > \underline{\mu}_b]
\end{aligned}$$

which is strictly positive as long as (58) is respected. □

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